Problem of the Month, November/December 2009

Please turn all solutions into Dr. Dunn's office, JB 322. You may slide your solutions under his door as well. Most elegant solution wins a $10 gift certificate to the bookstore! Solutions will be accepted anytime before or during finals week. Good luck!

We define a sort of differentiation on the real numbers as follows. Define the following operation $\partial : \mathbb{R} \to \mathbb{R}$ as satisfying *only* the following two properties.

1. For any integer n, we have $\partial(n) = 0$.
2. The product rule holds. That is, for any real numbers a and b, $\partial(a \cdot b) = \partial(a) \cdot b + a \cdot \partial(b)$.

Clearly, by property (1), every integer gets sent to 0 by ∂. The $10 prize goes to the submission that can find, with proof, the largest subset of the real numbers that must also get sent to 0 by ∂.