Problem of the Month, April 2009

Please turn all solutions into Dr. Dunn's office, JB 322. You may slide your solutions under his door as well. Most elegant solution of this month's problem wins a $10 gift certificate to the bookstore! Solutions will be accepted anytime during the month of April, 2009. Good luck!

Let $f(x)$ be a polynomial with real coefficients of degree n such that $f(x) \geq 0$ for all $x \in R$. Define $g(x) = f(x) + f'(x) + f''(x) + \ldots + f^{(n)}(x)$. Show that $g(x) \geq 0$ for all $x \in \mathbb{R}$.