Name: ______________________________

STOLEN Math 618 Final Exam of Awesomeness!!!

Directions: Please take this exam without cheating. It's worth 180 points, 30 points each. Please also read the directions to each question carefully–questions that you do not follow the directions on may result in a score of zero points for that question. You may use a writing utensil, a calculator, and your brain. Oh, and ROCK ON!!!

1. Please do exactly one of the following.

   (a) Suppose $f : \mathbb{R}^n \to \mathbb{R}^m$, and let $a \in \mathbb{R}^n$. Please define exactly what it means for ....

   (b) Please carefully state the .... theorem.

   (c) Please carefully state the .... theorem.

   (d) Let $f : U \to \mathbb{R}$, where $U \subseteq R^n$.

      i. Please carefully define the .... of f.

      ii. Please give conditions on the Jacobian of f that force ....

      iii. Suppose U is compact. Please ....

2. Please do exactly one of the following.

   (a) Suppose f is differentiable, ... Please do both of the following.

      i. Prove that the derivative of f is ....

      ii. Let $f(x, y) =$ ....

   (b) Suppose R is a rectangle, $a \in R$, and $f : R \to \mathbb{R}$ is continuous. Suppose .... subrectangle $S \subseteq R$ with ....

   (c) Suppose $f : \mathbb{R}^n \to \mathbb{R}^m$ is .... Please do both of the following.

      i. Show that ....

      ii. .... by the limit process, that .... for any $\alpha \in \mathbb{R}$.

      iii. ....

3. Please do exactly one of the following.

   (a) Let $F(x, y) =$ ...., and let ....

      i. Please verify that .... implicitly defines y as ....

      ii. Please compute $D\varphi(x)$ in ....

   (b) Suppose $S, T : \mathbb{R}^n \to \mathbb{R}^m$ are a linear maps. Please do all of the following.

      i. Please define the ....

      ii. Suppose $g : [0, 1] \to \mathbb{R}^n$ is continuous. Please show that ....

      iii. Prove .... strict.

   (c) Let $f(x, y) = (x + y, x - y)$.

      i. Please demonstrate that .... g.

      ii. Please .... Dg.

      iii. Find.................................... Dg ....

      iv. ....

4. Please do exactly one of the following.

   (a) Let $f : R \to \mathbb{R}$ be a rectangle, and suppose f is ....

i. .... "convenient criterion"......................................

ii. Suppose $R = [0,1] \times [0,1] \subseteq \mathbb{R}^2$. Let $\mathcal{P}_n^x$ be.....................................$[\frac{i-1}{n}, \frac{i}{n}] \times [0,1]$, for $i = 1, \ldots, n$. Let $\mathcal{P}_n^y$ .... Suppose there exists a number I so that for every n, we have .... Please prove that f is integrable.

(b) For this problem, ... *cubical norm.*

i. .... define $||x||_\square$, and describe ....

ii. Please prove that, if $||x||$ is the standard norm of the vector x, that ....

iii. Please carefully define the cubical operator norm $||T||_\square$, where $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear map.

iv. Please prove that ....

(c) Please prove that if $\alpha \in \mathbb{R}$, R is a rectangle, and f is integrable on R, ...

5. Please do exactly one of the following.

(a) Please do all of the following.

i. .... *volume zero.*

ii. Let $f(x) = x^2$. Please show that the set .... has volume zero.

iii. Please show that if X is a closed set of volume zero, then ....

(b) Please do all of the following.

i. Please construct functions $x(r,\theta)$ and $y(r,\theta)$ that parameterize $\mathbb{R}^2$ with polar coordinates.

ii. Please verify that the change of variable theorem holds for an integrable function f over ....

iii. ....

$$\int_A \sqrt{x^2 + y^2}\, dV.$$

(c) Please do both of the following.

i. Suppose f and g are integrable functions on a rectangle $R \subseteq \mathbb{R}^n$, and $g \leq f$. Prove that ....

ii. Suppose Ω is a region, and .... on Ω. Please find real numbers $I_* > -\infty$ and $I^* < \infty$ so that ....

6. Please do exactly one of the following.

(a) Suppose that f is integrable on the rectangle $R = [-1,1] \times [-1,1]$. Suppose also that for all $x \in R$, we have .... Prove that $\int_R f dV = 0$.

(b) ....

(c) Please establish the .... for integrals. That is, ... smooth path between them. Prove that there exists a $c \in \Omega$ so that ....