Hi everyone!!! Batman here. I've broken into Corey's office trying to steal your exam, but he caught me in the act and physically overpowered me before I was able to take all of it. But, here's what I was able to see. It appears that Corey was in the process of proofreading this exam before sending it to get printed. So, what I was able to steal is within ϵ of the actual test, in fact, ϵ is very small. I think he decided to stop for the night before checking for any typos, and that's when I came in, so that any differences between what you read below and the actual test will not be mathematical. Good luck, and ROCK ON!

Math 618 Midterm of Awesomeness

Directions: Please take this exam without cheating. It's worth 100 points. Please also read the directions to each question carefully–questions that you do not follow the directions on may result in a score of zero points for that question. Please be legible!!! If I can't read what you're writing, I won't be able to understand your argument! *You may use a writing utensil, a calculator, and your brain. Oh, and* ROCK ON!!!

1. (25 points) Please do exactly one of the following.
   (a) Suppose $f : \mathbb{R}^n \to \mathbb{R}^m$, and let $a \in \mathbb{R}^n$. Please define exactly what it means for f to be ....
   (b) Suppose .... and $f : \mathbb{R}^m \to \mathbb{R}^\ell$, $a \in \mathbb{R}^n$. ....
   (c) Let $f : \mathbb{R}^n \to \mathbb{R}$, and $a \in \mathbb{R}^n$. Suppose also that f is $\mathcal{C}^2$. Please carefully define the .... Please also define its associated ...., and show that if $n = 2$, then ....

2. (25 points) Please do exactly one of the following.
   (a) Let f be differentiable at a, $f : \mathbb{R}^n \to \mathbb{R}^m$. Let $v, w \in \mathbb{R}^n$, and $\alpha, \beta \in \mathbb{R}$. Prove ....
   (b) Suppose f is differentiable, $f : \mathbb{R}^n \to \mathbb{R}^m$. Please do both of the following.
      i. .... unique.
      ii. Let $f(x, y) = (2xy + x^2, \cos x - 6y)$. Show that .... $Df(x, y)$ for any (x, y).
   (c) Suppose $f : \mathbb{R}^n \to \mathbb{R}^m$ is differentiable at a. Please do both of the following.
      i. Show that .... for all $v \in \mathbb{R}^n$.
      ii. Suppose that $m = 1$, and ....

3. (25 points) Please do exactly one of the following.
   (a) .... Please do all of the following.
      i. Please define the ...., and prove that it exists. Any claims of continuity should be discussed, not just stated.
      ii. Show that for any $x \in \mathbb{R}^n$, we have $||Tx||$ ....
      iii. ....
   (b) Let $f(u, v) = u + 2v$, and let $g(x, y) = (u, v) = (x^2 + 2y, \sin y)$. Please do all of the following.
      i. .... g is differentiable at a, and that f is .... $g(a)$.
      ii. Please compute .... will probably have x, y, u, and v in it.

iii. Please actually compute what ....

iv. .... Draw a smiley-face on your paper at this point if they agree. If not, then please think seriously about checking your work.

(c) Suppose $f : \mathbb{R}^2 \to \mathbb{R}$, and suppose $a \in \mathbb{R}^2$. .... harmonic, that is, ... then a is not ....

4. (25 points) Please do exactly one of the following. You might consider doing this problem on the back of this page.

   (a) Let .... The answer to this question is more detailed than "we have a result which says we can do it." I would like you to produce the construction in that proof.

   (b) Let A be a square matrix. Define the exponential of A by the usual power series

$$e^A = \sum_{k=0}^{\infty} \frac{1}{k!} A^k.$$

   ....

   (c) Please do all of the following. For this problem, the objects A_k are square matrices for each k, and the objects v_k are vectors in $\mathbb{R}^p$ for each k. To distinguish between the euclidean length of a vector in $\mathbb{R}^p$, and the operator norm of a square matrix, we denote the operator norm of a matrix $||A_k||_o$, and the euclidean norm of a vector $||v_k||_e$. Recall that we may view a matrix as a vector in $\mathbb{R}^p$ for $p = n^2$, and so $||A_k||_o$ and $||A_k||_e$ both make sense.

      i. .... Use ϵ's and N's.

      ii. Notice you're using the euclidean norm above. Generally, one defines convergence of .... operator norm.

      iii. Please prove that these notions of convergence are ....

   (d) Let .... Also, give an example where X is not closed, f is continuous, $\{v_k\}$ is convergent (to a point not in X), but...............................

5. For 2 free points, tell me something funny!