Information on the Portfolio Assignment

November 18, 2009

Here is a list of problems that you may do to satisfy your portfolio requirement in this class, and as one of your portfolio entries with the M.A. program. Of course, you don't NEED to submit such an entry to the graduate school, but the format I'm requiring for this course is as such. You may choose any one of the following "problem" type problems, or "theorem" type results. Generally, for a portfolio entry, you must submit not only the problem and solution, or, statement of the theorem and its proof, you must also describe how the statement or proof is set in context within the class, and be sure to define any mysterious symbols–this might take place in some sort of preamble to your work that you write yourself–this information and preamble is part of your assignment for this class. For example, you might not have to type out what it means to be differentiable, although you may want to mention what it means to be $\mathcal{C}^1$. Another example: you may want to describe to the reader that $||T||$ is the operator norm of T and define it in this preamble portion of your submission. Finally, for brevity, some of the submissions can be found in the book and are referenced as such below–should you choose any one of those for your portfolio submission please actually type out the result on what you turn in, don't put something like "Theorem 1.7 on page 34. Proof: . . . ."

This document will outline those problems or Theorems suitable for our work prior to Chapter 7. There may be another document which outlines good problems or Theorems to do in Chapter 7 and beyond that will appear later.

The last piece of info I can offer is that these results should be in your own words as you understand them. Sure, a lot of these facts and their proofs or solutions may be found in the book, but blatant copying of the arguments found in the book or other obvious sources is not allowed. Instead, for example, this is your chance to expound upon a point that the book may consider obvious but that you don't. Or, you may find a different proof altogether, or a more efficient way of approaching a particular part of a proof. this exercise asks you to internalize the information and present it in your own words. Good luck!

1 Examples and Problems

1. Let $a \in \mathbb{R}^n$, let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear map, and let $v \in \mathbb{R}^n$. Please compute the directional derivative $D_vT(a)$. Please also show that T is differentiable at a, and compute $DT(a)$.

2. Please prove Proposition 2.3 of Chapter 3, page 92.

3. Please construct a function $f : \mathbb{R}^2 \to \mathbb{R}$ which has partial derivatives which exist, but that f is not differentiable.

4. Please do 3.2.14, page 97.

5. Please demonstrate, using the Maximum Value Theorem, that the operator norm of a linear map exists. Then, please prove Proposition 1.3 of Chapter 5, page 199.

6. Please do 5.1.5, page 201.

7. Please do 5.1.6, page 201.

8. Please do 5.3.2, page 215.

9. Suppose that $A : \mathbb{R}^n \to \mathbb{R}^n$ is a symmetric matrix, and that $Q(x) = x^TAx$ is its associated quadratic form. Using the constraint $g(x) = ||x||^2 - 1 = 0$, please use Lagrange multipliers to show that any constrained extrema of Q is a (unit) eigenvector of A.

10. Please do 6.1.6, page 249.

11. Please do 6.1.7, page 249.

12. Please do 6.2.2, parts (a) and (b), page 259.

13. Please do 3.c on the exam. There was a subtle misprint on the exam which it was hoped you would assume: you may assume the f is $\mathcal{C}^2$.

14. Please do 4.b on the exam.

15. Please do 4.d on the exam.

2 Theorems

1. Please prove 2.b.i on the exam, and please also prove Proposition 2.1 (page 88) of Chapter 3. (with this submission, it is appropriate to reference the definition of what it means to be differentiable in your preamble.)

2. Please prove Proposition 2.2 of Chapter 3, page 91.
3. Please prove Proposition 2.4 of Chapter 3, page 93.
4. Please prove Theorem 1.1 of Chapter 5, page 197.
5. Please prove the Maximum Value Theorem, Theorem 1.2 of Chapter 5, page 199.
6. Please prove the Uniform Continuity Theorem, Theorem 1.4 of Chapter 5, page 200.
7. Please prove that if A_k is a sequence of linear maps, and that $\sum_{k=1}^{\infty} ||A_k||$ converges, then $\sum_{k=1}^{\infty} A_k$ converges. (Here, $||A_k||$ is the operator norm of A_k.)
8. Please prove the Contraction Mapping Principle, Theorem 1.2 of Chapter 6, page 245.