

# Math 618 Midterm Exam Review # 1

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*Hello, humans from the 21st century! I took the liberty of looking at some of the material that you've been covering, and, with the use of a 31st century ray-gun, I'm able to peer into Corey's skull to see what he thinks would be good to study for, section by section, in preparation for your upcoming exam. Good luck! And, ROCK ON!*

1. Section 3.1: Partial derivatives and directional derivatives. Here, we have been interested in mostly reviewing what we already know from multivariable calculus insofar as partial and derivatives are concerned. Emphasis, of course, is places on the higher-dimensional setting. The definition of each is important to know, of course, along with the ability to do any example. Also, the understanding that  $D_v(f)(a) = \nabla f(a) \cdot v$  has been useful.
2. Section 3.2: Differentiability. We built the analogue of what the derivative of a function  $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$  should be (or, at least, at a point  $a$  in an open subset  $U$  of  $\mathbb{R}^n$ .) Knowing the exact definition of what  $Df$  is and that it's unique will be important, not just Proposition 3.2.1 (page 88) which states that if  $f$  is differentiable then  $Df$  must be the Jacobian matrix. Each of Proposition 2.2, Proposition 2.3, and especially Proposition 2.4 will be useful.
3. Section 3.3: Differentiation rules. Here we introduced ourselves to the operator norm  $\|T\|$  of a linear map  $T$ , and waited until later to convince ourselves that this quantity

actually exists. In fact, there are basic facts about the operator norm that will be tested upon, in addition to being part of what could be a larger problem involving these norms—I would know all of these basic facts and their proofs as they provide a technical backdrop to our studies... this includes exercises 5.1.5 and 5.1.6 on page 201. Besides that, the basic differentiation rules when phrased in the higher-dimensional setting are studied here, and it's important to know them, and their proofs.

4. Chapter 4: Linear Algebra review. This chapter wasn't explicitly covered in class, but I felt that it would be good to remind you that basic linear algebra that you've seen Corey use in class will be knowledge that is assumed that you know throughout the course. So, if there have been any nagging linear algebraic issues that have been bothering you, this chapter may be a good place to look to fill in those gaps. Other than that, nothing specifically from this chapter will be asked, it will be assumed that you know a certain amount of linear algebra that will help you solve our other problems.
5. Section 5.1: Compactness and the Maximum Value Theorem. This section kicks off our study of extrema and their possible locations. The Maximum Value Theorem 1.2 and its preliminary supporting result Theorem 1.1 (pages 197 and 199) are essential (Theorem 1.2 more so). It's Theorem 1.2 that insists that the operator norm exists, among other maximum or minimum value questions. Although uniform continuity was not emphasized, it would still be good to know and understand Theorem 1.4 as an application of continuity to compact domains.
6. Section 5.2: Maximum/minimum problems. Here, Corey did a poor job of accurately defining local extrema, but he's going to pretend he wrote it perfectly and that you can compare what you wrote with how the book defines it. Lemma 2.1 is essential as it characterizes local extrema as critical points (if the extrema is at a place where the function is differentiable, then the derivative vanishes). So, an understanding of these processes is essential for determining how to find extrema—one of our goals.
7. Section 5.3: The second-derivative test. This great section introduced us to the wonderful harmonious relationship between matrices and their “associated quadratic forms”. Namely, if  $A$  is a square matrix, then it's associated quadratic form  $\mathcal{A}(x) = x^T A x$ . We apply this construction to the Hessian of a function and prove the second derivative test. All definitions and theorems are of interest, except the stuff from Example 3 on (that's the stuff we covered in class).
8. Section 6.1: The contraction mapping principle. This section deals with the prerequisites needed for the inverse function theorem (and thus, the implicit function theorem as well). The discussion about infinite series of vectors is relevant on its own, and then subsequently its application to the Contraction Mapping Principle Theorem 1.2. Homework numbers 6 and 7 had been assigned to emphasize the difference

between the norm of a vector in Euclidean space (using the Euclidean norm) versus the norm of an operator (the operator norm). In particular, Proposition 1.1 may only be used with the Euclidean norm. Any thoughts on whether or not it would hold using the operator norm? Any thoughts on whether or not that would be a good exam question? The Mean Value Inequality Proposition 1.3 is also used wildly in our subsequent work and is worth knowing.

9. General suggestions. This exam is going to be awesome! Generally speaking, the exam will be a collection of homework problems, results we proved in class, and problems that Corey has made up to test your knowledge. That means a firm understanding of those problems and results is key, in addition to an ability to incorporate that knowledge to new situations. Knowing definitions and statements of results *verbatim* will be extremely important, as some of the test may ask you to produce exactly that. This guide is a rough idea as to how to focus your thoughts. Other than that, the exam should be a fun ride! Good luck, and ROCK ON!!!!