Math 610 Homework # 4

By: An Advertisement for Coffee

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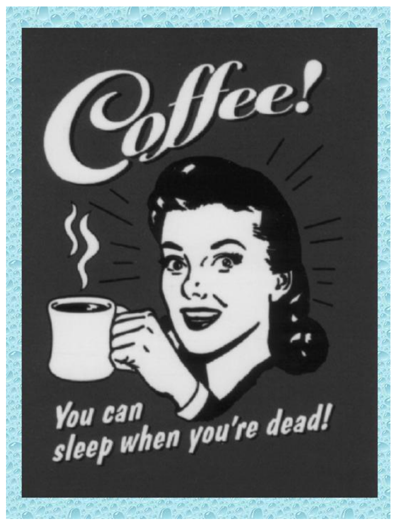

The following assignment will help us immensely in determining the simplicial homology groups of Δ-complexes, along with some other odds and ends about singular homology in general. Coffee also helps in this process, as Corey has discovered over the years. We, the coffee corporations of America can now rejoice that another human has fallen prey to the sinful life of coffee drinking! We're trying to capture you all as well, but we've had our eyes on Corey for a while. Go ahead, dare him to go a day without coffee!!! This assignment will be due May 23. ROCK ON!!!

1. ($\star$) Show that if $X \simeq Y$, then $H_n(X) \cong H_n(Y)$. You may assume that the homotopy property holds: if $f \simeq g : X \to Y$, then $f_* = g_*$.

2. Use the above and a previous homework assignment to prove that if A is a deformation retract of X, then $H_n(X) \cong H_n(A)$.

3. ($\star$) We know that if X is path-connected, then $H_0(X) \cong \mathbb{Z}$. Let (P) be a generator for this homology group, i.e., $H_0(X) = \mathbb{Z} \cdot (P)$. Let (Q) be any other zero-simplex

in X. Prove that (Q) is also a generator for $H_0(X)$. That is, show (P) and (Q) represent the same equivalence class in $H_0(X)$ by showing $(P) - (Q) \in Im\partial_1$.

4. $(\star)$

   (a) Exhibit a Δ complex structure on S^1 and compute its Δ complex homology $H_n(S^1)$.

   (b) Repeat the previous problem, except use a different Δ complex structure. The math Gods have been talking with the coffee drinkers of America and we all agree that the answers you get should be the same for both parts (a) and (b).

5. $(\star)$ Exhibit a Δ complex structure on the torus $\mathbb{T}^2$ and compute its Δ complex homology $H_n^{\Delta}(\mathbb{T}^2)$.

6. $(\star)$ Exhibit a Δ complex structure on the Klein bottle $\mathcal{K}$, and compute the Δ complex homology $H_n^{\Delta}(\mathcal{K})$.