# Math 610 Homework # 2

The IRS

April 16, 2007

*Hello taxpayers! This is the IRS here to point out that your taxes are due very soon. In fact, by the time you get this assignment that we and the government have put together for you, your taxes could already be late! Ha ha ha ha ha!!!! April 15th is on a Sunday, so you might be okay if you sent in your taxes later than that, but we would advise that you not do that. Besides the wonderful homework questions we've listed below, we sort of also want to know something: Who does the IRS report its earnings to? In any event, this homework assignment will be due Wednesday, April 25.* ROCK ON!

1. Suppose that $X = A \cup B$, a topological space. Suppose also that $A$ and $B$ are closed subspaces of $X$ (both endowed with the subspace topology). Let $f : X \to Y$ be a function so that $f$ is continuous when restricted to each of $A$ and $B$. Prove that $f$ is continuous as a function $f : X \to Y$.

2. A *discrete map* is a continuous function $d : X \to D$, where $D$ is a finite set given the discrete topology. Prove that $X$ is connected if and only if every discrete map is constant.

3. Prove that the space consisting of two points $X = \{p, q\}$ with topology given as having open sets: $\emptyset, \{p\}, \{p, q\}$ is path connected.

4. A *locally path connected* space $X$ is a space satisfying the following property: For all $x \in X$ there exists an open path connected set $U$ containing $X$. Suppose that

$X$ has finitely many path components. Prove that each of the path components are both open and closed. (*The IRS says on Form 234534534XX2345 that it thinks that it's not needed to assume that $X$ has finitely many path components and that this is still true for arbitrarily many path components. But does anyone really look at those forms anyway?*)

5. $\star$ The following sequence of facts will be used to prove *The Fundamental Theorem of Point-Set Topology*. Recall that the definition of a Hausdorff space appeared on homework assignment #1.

   (a) If $Y$ is a Hausdorff space, then any compact subset of $Y$ is closed.

   (b) If $X$ is compact and $f : X \to Y$ is continuous, then $f(X)$ is compact.

   (c) Any closed subset of a compact space is compact.

   (d) (The Fundamental Theorem of Point-Set Topology) If $X$ is compact, $Y$ is Hausdorff, and $f : X \to Y$ is continuous, one-to-one, and onto, then $f$ is a homeomorphism.

6. Suppose $f : X \to Y$, a continuous map of topological spaces. The map $f$ is called an *open* map if for every open set $U \subseteq X$, the set $f(U)$ is open in $Y$. The map $f$ is called *closed* if for every closed set $F \subseteq X$, the set $f(F)$ is closed in $Y$.

   (a) Assume that $f$ is a bijection. Show that an open map is a homeomorphism.

   (b) Assume that $f$ is a bijection. Show that a closed map is a homeomorphism.