STOLEN Math 555 Final Exam of Awesome goodness

Oh my gosh!!! The final exam has partially been stolen by a masked human! The picture above is my attack kitten catching the burglar in the act. The very next instant, of course, the kitten went crazy on this guy, wrestled the guy away from him, and tore his face and hands to shreds! But he got away, and this is what he managed to steal. Please also understand that small changes will be made to the version that was stolen, Corey was going to proofread it for errors when all of this went down. ROCK ON!!!

1. (25 points) Please choose exactly 2 of the following to do. No, you won't get any extra credit if you do more.
   (a) Let X be a set. Let τ be a collection of subsets of X. Please carefully define ....
   (b) Let $\mathcal{B}$ be a collection of subsets of X............................
   (c) ....
   (d) Please carefully define what it means for a function ....
2. (25 points) Please choose exactly 2 of the following to do. No, you won't get any extra credit if you do more.
   (a) ....
   (b) .... *compact.*
   (c) Let $\sim$ be an equivalence relation on a topological space X. Please carefully define .... on the set $X/\sim$.
   (d) Please carefully state ....
   (e) Please carefully define...........................
3. (25 points) Please choose exactly one of the following to do. No, you won't get any extra credit if you do more.
   (a) Suppose that $A \subseteq X$, and that for all $x \in A$, there exists a neighborhood U_x ....
   (b) Let X be a Hausdorff space, and let $x_0 \in X$. ....
   (c) Create an example of a topological space X with nonempty subspaces A and Y so that $A \subseteq Y \subseteq X$, subject to each of the following conditions (not simultaneously, create a new example for each part below):
      i. A is open in Y,...........................
      ii. A is closed in Y, Y is neither closed or open in X, ...
      iii. Prove that if A is closed in Y, and Y is closed in X, then A is ....

4. (25 points) Please choose exactly one of the following to do. No, you won't get any extra credit if you do more.

   (a) Let $A \subseteq X$. Prove that if $x \in \bar{A}$, then ....

   (b) Let $A \subseteq X$, and let $x \in X$. .... Prove that $x \in \bar{A}$.

   (c) Please do both parts.

      i. Construct a topological space X which is ....

      ii. Please prove that if $Y \subset X$, and X is Hausdorff, then ....

   (d) Let $A =$ .... Please compute the set A', and $\bar{A}$.

5. (25 points) Please choose exactly one of the following to do. No, you won't get any extra credit if you do more.

   (a) Please do both of the following parts.

      i. Suppose $f : X \to Y$ is continuous, and that X is compact. Please show ....

      ii. Let $X = \{p, q\}$, with topology $\tau = \{\emptyset, X, \{p\}\}$..........................

   (b) Please do both of the following parts.

      i. Suppose X is connected, and $f : X \to Y$ is continuous. ....

      ii. Please show that if X is path connected, then X is ....

   (c) Let F be a closed subset of a compact space X. ....

   (d) Please do both of the following parts.

      i. Both of the properties "compact" and "connected" are homeomorphism invariants. .... and prove it.

      ii. The separation axioms T_3, and T_4 are homeomorphism invariants. .... and prove it.

6. (25 points) Please choose exactly one of the following to do. No, you won't get any extra credit if you do more.

   (a) Let $f : X \to Y$ be a bijection. Please do both of the following parts.

      i. ....

      ii. .... a closed map.

   (b) Assume that all of the questions I've asked you on this test are actually true statements..........................

   (c) Please do both of the following parts. Let X be a Hausdorff space, and let A be a compact subset of X.

      i. ....

      ii. Prove that any compact subset of a Hausdorff space is closed.

   (d) Prove that $X \times Y$ is compact if and only if ....

7. (25 points) Please choose exactly one of the following to do. No, you won't get any extra credit if you do more.

   (a) Suppose $K \subseteq \mathbb{R}^n$. Please carefully show that K is compact if ....

   (b) Please show that a compact Hausdorff space is ....

   (c) Suppose $A \subseteq B \subseteq \bar{A}$. ....

   (d) Please do both of the following parts.

      i. Please give an example of an topological space (where X is infinite) that has a point $x_0 \in X$ so that ....

      ii. Please determine whether or not $\mathbb{R}$ with the finite compliment topology is ...., and prove your claim.

8. (25 points) Please choose exactly one of the following to do. No, you won't get any extra credit if you do more.

   (a) Please show that $[0, 1]$ is connected. (Do not use a path-connected argument).

   (b) Let $B_r(0) =$ .... be .... the origin. Please prove or disprove: If $0 < r < 1$, then $B_1(0)/B_r(0) \approx B_1(0)$.

   (c) Suppose X is a normal space, and A and B are disjoint closed subsets of X. Please prove........................... $U_0, U_{1/2}$ and U_1 with $A \subseteq U_0$, $B \subseteq U_1^c$, $\bar{U}_0 \subseteq U_{1/2}$, and $U_{1/2} \subseteq \bar{U}_1$.

   (d) ....

   (e) Please do parts (i), (ii), and (iv) below, but do not do part (iii):

      i. ....

      ii. ....

      iii. Show that $\mathbb{R}^n$ is not homeomorphic to $\mathbb{R}^m$ for positive integers $m \neq n$.

      iv. ....

9. For 2 free points, tell me something funny!