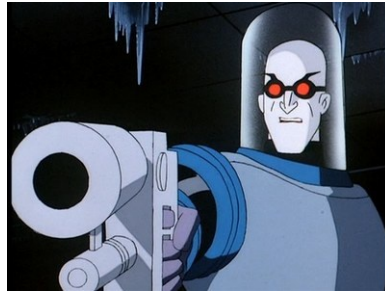


Stolen exam!!!



Hello, San Bernardino area residents, I am Mr. Freeze. I was hired to steal the Math 555 exam to give it to you as a guide to studying it. Of course, my body needs to be kept quite cold, and because it's so ridiculously hot here I could only get a portion of the exam before retreating to my secret cave of ice located in the frozen foods isle at the 40th Street Stater Brothers. Good luck. ROCK ON!!!

1. (20 points) Please choose exactly two of the following to do. No, you won't get extra credit by doing more.
 - (a) Let X be a set, and let τ be a collection of subsets of X . Please carefully define what it means for
 - (b) Let $Y \subseteq X$, where X is a topological space. Please carefully define.....
 - (c) Please carefully define the
 - (d) Let \mathcal{B} be a collection of subsets of a topological space X
 - (e) Let $A \subseteq X$. Please carefully define two sets: the.....

2. (20 points) Please choose exactly one of the following to do. No, you won't get extra credit by doing more.
 - (a)open set U_x with $x \in U_x$ and $U_x \subseteq A$. Prove.....
 - (b) Formulate closed sets Prove that
 - (c) Suppose there is a topology on \mathbb{R} for which the intervals $[a, b)$ and $(a, b]$ are open sets for all real numbers $a < b$

3. (20 points) Please choose exactly one of the following to do. No, you won't get extra credit by doing more.
 - (a) so that $\overline{\{x_0\}} = X$.
 - (b) Suppose there exists a nonempty proper subset $U \subsetneq X$ which is both open *and* closed. Prove that there exists a nonempty and open set V so that
 - (c) Define the projection $\pi_2 : X \times Y \rightarrow Y$ by $\pi_2(x, y) = y$. Endow $X \times Y$ with the product topology.
 - (d) Prove that the set $\mathcal{B} = \{(a, b) | a, b \in \mathbb{Q}\}$ is a basis for the standard topology on \mathbb{R} .

4. (20 points) Please choose exactly one of the following to do. No, you won't get extra credit by doing more.
 - (a) Please do both of the following parts:
 - i. Construct a topological space which is not.....
 - ii. Prove that any subspace of a Hausdorff space is

- (b) Let X be a Hausdorff space, and let $x_0 \in X$. Prove that $\{x_0\}$ is closed.
- (c) Let $A, B \subseteq X$
- (d) Please do 3 of the following 5 questions. (If you choose to do this one, I do not recommend you try the last one on the list below, I put it there as more of a challenge.) Use the standard topology on \mathbb{R} for these questions. Justify your answers with short proofs.
 - i. Give an example of a subset A for which $A' \subseteq A$.
 - ii. Give an example of an infinite subset A
 - iii.
 - iv.
 - v. Is it possible that $A' = \emptyset$?

5. (20 points) Please choose exactly one of the following to do. No, you won't get extra credit by doing more.

- (a) Let $A \subseteq X$, and let $x \in \bar{A}$. Prove that if U is a neighborhood of x ,
- (b) Let $A \subseteq X$. Suppose $x \in X$, and that whenever U is a neighborhood of x , then $U \cap A \neq \emptyset$. Prove that
- (c) Let $A \subseteq X$. Suppose that the topology on X is given by the basis \mathcal{B}
- (d) (This problem deals with sequences in topological spaces, and I recommend against choosing this one to do.) Choose one of the following to do:
 - i. This notion
 - ii. Construct a topological space X and..... I strongly suggest you look elsewhere, such as a non-Hausdorff space.

6. For 2 free points, tell me something funny!