HW # 6

The San Diego Chicken

December 3, 2008

(He's a mascot, he doesn't say much... but this assignment will be due on the day of the final exam). ROCK ON!

1. Please do questions 1(a), 2, 3, 5 and 6 from Section 26 (pages 170–171).
2. Suppose K is a subset of $\mathbb{R}^n$. Show that K is compact if and only if K is closed and bounded.
3. Show that the properties of first and second countability are invariant under homeomorphism. That is: suppose $f : X \to Y$ is a homeomorphism. Show that X is first countable if and only if Y is first countable. Show also that X is second countable if and only if Y is second countable.
4. Show that the separation axioms T_2, T_3, and T_4 (Hausdorff, Regular, and Normal) are homeomorphism invariants. That is, for $k = 2, 3$, and 4, show that if $f : X \to Y$ is a homeomorphism, then X is T_k if and only if Y is T_k.
5. Show that every compact Hausdorff space is normal.