

# HW # 5

Pinky and the Brain!

November 13, 2008



*Hello unwilling participants of my world domination scheme! I was in the middle of telling Pinky about what we're going to be doing tomorrow night when I was rudely interrupted by Corey asking me to write this homework assignment for you. So here! It's due Tuesday, November 25th. ROCK ON!*

1. Please do numbers 1, 2, 4 and 5 on page 152. Then do parts (a) and (c) of number 1 of page 157, and numbers 2 and 3. Corey stresses that you'll more than likely have to use the intermediate value theorem for 2 and 3, and for number 2, the definition of  $S^1$  is  $S^1 := \{z \in \mathbb{C} \mid |z| = 1\}$ .
2. For the following, let  $f : X \rightarrow Y$  be a bijection of sets.
  - (a) Since  $f$  is a bijection,  $f^{-1}$  exists. Denote  $g = f^{-1}$ . Prove that for all  $U \subseteq X$ , that  $f(U) = g^{-1}(U)$ .
  - (b) Suppose  $f$  is continuous. Prove that  $f$  is a homeomorphism if and only if  $f$  is an open map (recall that an open map is a function  $f$  so that  $f(U)$  is open whenever  $U$  is open).

- (c) Suppose  $f$  is continuous. Prove that  $f$  is a homeomorphism if and only if  $f$  is a closed map (recall that a closed map is a function  $f$  so that  $f(F)$  is closed whenever  $F$  is closed).
3. DISprove the following assertion: Let  $\tau$  and  $\tau'$  be two topologies on the same space  $X$ . Then if  $(X, \tau) \approx (X, \tau')$ , then  $\tau = \tau'$ .
  4. A *discrete map* is a continuous function  $d : X \rightarrow D$ , where  $D$  is a finite set given the discrete topology. Prove that  $X$  is connected if and only if every discrete map is constant.
  5. Prove that the space consisting of two points  $X = \{p, q\}$  with topology given as having open sets:  $\emptyset, \{p\}, \{p, q\}$  is path connected.
  6. By definition, a *starlike* space  $X$  is a subset of  $\mathbb{R}^n$  for which there exists a point  $a \in X$  so that for all  $x \in X$ , there exists a straight line path  $\gamma_x : [0, 1] \rightarrow X$  so that  $\gamma_x(0) = x$  and  $\gamma_x(1) = a$ . Prove that any starlike space is path-connected. (I think you'll all see how to connect any two points in such a space, but I think a fact which is frequently overlooked would be to establish that such a path be continuous.)