Math 555 Homework # 3 Solutions

A confident Zoidberg

October 29, 2008

Here are the solutions to homework # 3! Enjoy! Rock on!

2 Suppose A is closed in Y and Y is closed in X. We must show that A is closed in X. Since A is closed in Y, there exists a closed set F in X for which $A = F \cap Y$. Thus, as the intersection of two closed sets in X, A is closed.

3 Corey recalls discussing this one in class, and declines to type out solutions on it.

6 (a) Suppose $A \subseteq B$. We must show that $\bar{A} \subseteq \bar{B}$. So let $x \in \bar{A}$, and let U be a neighborhood of x. Then $U \cap A \neq \emptyset$. But $A \subseteq B$, and so $U \cap B \neq \emptyset$. Thus, by theorem 17.5, $x \in \bar{B}$.

(b) Corey recalls writing a complete solution on the board in class of this one.

(c) Let $x \in \cup_\alpha \bar{A}_\alpha$. The for some α, we have $x \in \bar{A}_\alpha$. Let U be a neighborhood of x. Thus, by theorem 17.5, we have $U \cap A_\alpha \neq \emptyset$. Thus, $U \cap (\cup A_\alpha) \neq \emptyset$. So again by Theorem 17.5, $x \in \overline{\cup_\alpha A_\alpha}$.

We construct an example where equality fails–this will not be difficult since infinite unions of closed sets need not be closed. Thus, we take $A_n = [-1 + \frac{1}{n}, 1 - \frac{1}{n}]$. Thus, $\bar{A}_n = A_n$ since A_n is closed for all n. But the numbers ± 1 are not in A_n for any n, and thus $\pm 1 \notin \cup \bar{A}_n$.

On the other hand, $\cup A_n$ has both ± 1 as a limit point, since every neighborhood around either of these points intersects some A_n at some point other than ± 1, and thus intersects $\cup A_n$ at some point other than ± 1. Since $\overline{\cup A_n} = \cup A_n \cup (\cup A_n)'$, we know that both $\pm 1 \in \overline{\cup A_n}$. So this is an example where $\overline{\cup A_n} \supsetneq \cup \bar{A}_n$.

11 Let X, Y be Hausdorff spaces, and let $(a, b) \neq (x, y) \in X \times Y$. Then one of the following is true: $a \neq x$ or $b \neq y$. Suppose without loss of generality (since both X and Y are Hausdorff) that $a \neq x$. Then there exists open neighborhoods U_a and U_x of a and x respectively, where $U_a \cap U_x = \emptyset$. Let V_b and V_y be any neighborhoods of b and y. Then the open sets $U_a \times V_b \cap U_x \times V_y = \emptyset$, since any elements in such an intersection would have X coordinate common to the sets U_a and U_x, of which there are no common elements.

12 Let $Y \subset X$, a Hausdorff space. Let $x \neq y \in Y$. Then since $x \neq y$ in X as well, and X is Hausdorff, then there exists neighborhoods U_x and U_y of x and y (respectively) which are disjoint. Then the sets $U_x \cap Y$ and $U_y \cap Y$, which are neighborhoods of x and y in Y are also disjoint.

20 Corey sort of thinks we talked about this one in class, but encourages you all to ask if you have any questions. ROCK ON!