Math 555 Homework # 2

Mr. Burns

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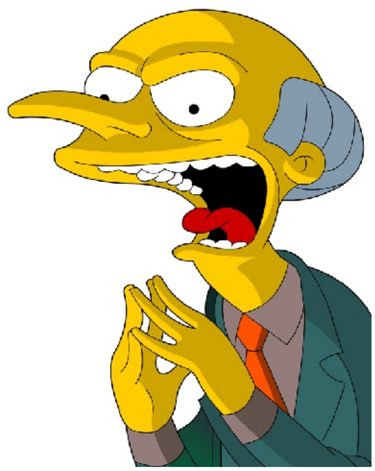

Hello, folks. Between naps I like to wake up and diabolically laugh. Sometimes I also write topology assignments as well. This assignment will be due on October 16th. HAHAHAHAHAAAAAA!

1. Starting from page 91, please do problems 1, 3, 4 and 6. Even though I'm a terrible boss who likes to see you all work your fingers to the bone proving topology questions, I have information about number 4 that will help you: in fact, I don't believe it is stated quite precisely enough. And so please see below for more information that will be helpful for problem number 4.

   For problem 4: The spaces X and Y are assumed to be topological spaces, and the topology on the product space $X \times Y$ is assumed to be the product topology. The maps $\pi_1 : X \times Y \to X$ and $\pi_2 : X \times Y \to Y$ are given as

$$\pi_1(x, y) = x, \quad \text{and } \pi_2(x, y) = y.$$

These maps are known as *projections*. You are asked to show that each of these are *open* maps in problem 4. . . that is, you're asked to show that each of these maps takes open sets to open sets.

2. (a) Suppose that X is a topological space, and that $A \subseteq Y \subseteq X$. Prove that if A is open in the subspace topology of Y, and Y is open in X, then A is open in X. The justifies the statement "open subsets of open subsets are open." Isn't that cute?

   (b) Suppose again that $A \subseteq Y \subseteq X$, and that A is open in Y. But do *not* assume that Y is open in X. Give an example of a situation where A is open, an example where A is closed, and an example of when A is neither open or closed. (This may require you to possibly choose different Y as well.)