HW # 1

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Hi kids. Brian Griffin here from Family Guy. I'm really an intellectual dog, not just a martini-drinking slacker. So, to prove it to the world, I thought I would come up with your first homework assignment. Here it is below. It'll be due on Tuesday, October 7th. ROCK ON!

1. Please do problems 1 and 8 on page 83.

2. Suppose X is a set, and τ is a collection of subsets of X so that (1) $X, \emptyset \in \tau$, (2) for any collection $U_\alpha \in \tau$, $\cup_\alpha U_\alpha \in \tau$, and (3) for any $U_1, U_2 \in \tau$, we have $U_1 \cap U_2 \in \tau$. Prove that τ is a topology on X. (This relaxes the condition of finite intersection to an easier-to-check alternative of checking that the *pairwise* intersection of open sets is open, rather than checking that any *finite* intersection of open sets is open).

3. Let X be a set, and let τ be a topology on X. Recall that a *closed* set is of the form $U^c := X - U$ for some open set U. I strongly suggest you use DeMorgan's Laws to prove the following three facts:

(a) X and $\emptyset$ are closed.

(b) For any collection F_α of closed sets, then $\cap_\alpha F_\alpha$ is closed.

(c) For any finite collection of closed sets F_i $(i = 1, 2, \ldots, n)$, then $\cup_{i=1}^n F_i$ is closed.

In fact, the opposite implication is true (which I don't require you to check, although it may be a good idea to do that for your own understanding). Thus, accordingly, topological spaces could very well have been defined by specifying *closed* sets rather than open sets–the same information is contained in either description.

4. Let $X = \mathbb{R}$. Suppose that there is a topology on X for which any set of the form $[a, b)$ and $(a, b]$ are open. Prove that this topology must coincide with the discrete topology (that is, this topology and the discrete topology on X have the same open sets).

5. Make up your own topological space. Be creative! (That is, specify a set X, and a collection of subsets τ, and prove your collection of subsets forms a topological space.)