

Exam # 2 Review Sheet of fun!!!!

Stewie and Brian

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Hi everyone, this is Brian here with a review sheet for your upcoming exam. Your exam will be on Wednesday, November 14th, and will cover the sections below. Even though I'm the family dog, I can speak and drink martinis and understand all of the stuff Corey has taught you. My troubled friend Stewie just likes to cause trouble. He and Corey are out playing in the sandbox and I'm stuck inside typing this review sheet for you. Oh well. I can still say, though, ROCK ON!!!!

1. Section 3.5: Bolzano-Weierstrass Theorems. We covered half of this section on the midterm, and what Corey expects you to know for this upcoming exam will be the Bolzano-Weierstrass theorem for sets. And, of course, you'll need to know something about accumulation points to understand this theorem. Those homework questions which you didn't hand in before from this section will be great practice, and I also suggest that you pose questions to each other that have to do with accumulation points. Do some thinking and try to come up with a great example or question, pose it to each other, and either answer it or try to have someone else explain it to you (or, of course, talk to me). These discussions and reflections really are the meat of the learning that takes place when you study analysis, and I encourage you all to try it!
2. Section 3.6: Cauchy sequences. I would know the definition of Cauchy sequences, as they are a very important notion. In addition, I would know that Cauchy sequences and convergent sequences (of real numbers) are one in the same (Theorem 3.12), and

to study the homework questions for more indications as to why Cauchy sequences can be fun to study (of course, the lecture notes will have more information along these lines, along with examples). Pay special attention to the homework as there seem to be some great questions there.

3. Section 3.7: Limits at Infinity. This section really just revises the definition of what it means to say $\lim_{n \rightarrow \infty} x_n = x$, where x could be $\pm\infty$. I would know these definitions as well, and be able to apply them.
4. Section 3.8: The limsup and liminf. This section starts out with a great observation: any sequence in \mathbb{R} has a subsequence with limit in $\mathbb{R}^\# = \mathbb{R} \cup \{\pm\infty\}$. Thus the set $E \subseteq \mathbb{R}^\#$ of subsequential limits of any sequence is nonempty, and thus we may well-define $\sup(E) = \limsup(x_n)$ and $\inf(E) = \liminf(x_n)$. It's great that these numbers always exist, and can help distinguish between two sequences (particularly if these are non-convergent sequences, where simply computing their limit wouldn't be a way to tell them apart). The consequences of these definitions are too numerous to list, but all of the examples and results in this section, in addition to the homework, will be great to know.
5. Section 4.1: Continuous Functions. This section really is just a primer to begin our study of continuous functions. There are basic definitions given, and a few basic results. What's more interesting is . . .
6. Section 4.2: Continuity and Sequences. There is a great theorem that phrases continuity of functions in terms of limits of sequences (Theorem 4.1). This is a great theorem that will have a great many uses. In particular, it will allow us to study a new object (continuous functions) in terms of old objects (sequences) which we have already studied.
7. Other comments. This exam will look a lot like the last one, same general format. Keep in mind that I leave town November 10th, and your exam isn't until the 14th. That means that you should plan ahead to ask Corey any questions you have either Wednesday in class (November 7th), or Thursday during his office hours (or by appointment). You will have a substitute to proctor the exam, so be sure to write very clearly and accurately, since the proctor will be able to provide very limited answers about the exam itself. Other than that, don't forget to ROCK ON!!!!