Exam # 1 Review Sheet

By: Sideshow Bob

Hello everyone, this is Sideshow Bob here to tell you about the upcoming exam. As you can see, I'm not very happy that the exam will be soon... I am a crazy homocidal maniac who would rather be killing Bart Simpson, but I can take time out of my busy day to give you some suggestions. Below I've listed what I find interesting from each section, and finish off with some more general information about the exam.

1. Section 3.1: Convergence. The concepts here are basic, but of utmost importance. We learn the definition of convergence of a sequence of real numbers and study examples. In particular, it is this definition of convergence which tells us how to establish convergence for every sequence. Later, we learn some shortcuts, but it is still important to know how to establish that a sequence converges using only the definition. The homework is a great tool to help you practice your skills and to see how well you know the material.

2. Section 3.2: Limit Theorems. This section made life a little easier, but we must remember that the theorems which help us out the most (Theorem 3.2) only may be applied when the sequences you're starting with converge. It is incorrect to conclude, for instance, that $\lim(x_n + y_n) = \lim(x_n) + \lim(y_n)$ without explicitly knowing that each of the sequences x_n and y_n themselves converge. The same goes for the rest of the theorems in this section. Other important theorems include Theorem 3.3, 3.4 and 3.5. Pay special attention to what the conclusions and hypotheses are in each theorem. For instance, in the Squeeze theorem, one concludes that a sequence converges from information about two other *convergent* sequences. In Theorem 3.3 one *begins* with a convergent sequence in $[a, b]$ and concludes that its limit must also be in $[a, b]$. If you didn't hypothesize that the sequence converges then you would only be able to conclude that a *subsequence* converges, and this is a fact not proved

until Section 3.5. Knowing what the theorems say is essential to being able to use them, and this is what we learn these theorems for, after all.

3. Section 3.3: Subsequences. We learned about subsequences here, and that if a sequence converges, then every subsequence must converge to the same limit. The homework also supply interesting facts which again test your knowledge.

4. Section 3.4: Monotone Sequences. This was a big section. We learned that any bounded monotone sequence converges, and that *any* sequence has a monotone subsequence. Also there were a bunch of examples of recursively defined sequences which we studied, and spent some time flushing out some of the technicalities of showing such sequences converge... see Example 3.15, and HW # 1–3). We also learned the Nested Intervals theorem, which will be of use later.

5. Section 3.5: Bolzano-Weierstrass Theorems. This section is pretty big, too. We can finally state one of the most important theorems in all of mathematics, which is that any bounded sequence has a convergent subsequence. The exam will include this theorem, but not anything past it, in particular, it will not cover accumulation points or the Bolzano-Weierstrass theorem for sets. This material WILL be on the next exam, though.

6. General suggestions and thoughts. The exam will have 5 questions. From your mathematical background, you know this statement has no actual content, since any of the 5 questions could have infinitely many parts. The first question has 2 parts and may ask you to define "convergence" as one of the parts. Questions 2–5 each have 2 parts. You are to do only one of the two parts for these questions, and these directions are clearly written on the test, so no need to fret. And, you don't get extra credit if you do BOTH of the parts when I only ask you to do one of the two. The questions 2–5 each deal with a different concept and it is debatable whether or not the level of difficulty increases as you go on. Corey does not intend for these questions to be difficult, but he isn't just going to make the whole test one big "can you memorize the proofs on the homework" challenge–after all, this isn't the point. As I mentioned in class, one of the main goals from this course is for you not just to learn Analysis, but to become more able to solve a variety of problems on your own, basing your solutions on rigor and getting started with your own creativeness. Thus, there will be questions you've never seen before, but every question Corey asks will have a solution which can easily be deduced from the material contained in the homework and the class notes. I hope this helps! Rock on!