

Thoughts on the 4th HW assignment

By: Corey's broken jump-shot

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Corey has been running around all day and has asked me to look at the homework and type up some thoughts, and these won't necessarily be the ones he grades. Since Corey will take me with him on his trip, these thoughts aren't meant to be proofs, but just suggestions.
ROCK ON!

1. #7 from 3.5: The only such sequences are eventually constant. To see this, simply consider the neighborhood around the isolated point that contains no other points of A . The sequence is eventually in this neighborhood, and thus must eventually be just that point.
2. # 3 from 3.6: Since Cauchy sequences converge, the sequence x_n is a sequence of integers that converges. We already showed in a very painful manner that all such sequences are eventually constant (previous HW problem). The other implication is more or less obvious, that all eventually constant sequences converge. But the key is noticing that Cauchy sequences and convergent sequences are the same family of sequences.
3. # 3 in Section 3.7: I think 3(a) we did in class, but if we didn't, then it follows from the definition. Just remember that you've been told that $x_n < 0$ in this case. For part (b) consider $x_n = (-1)^n n$.
4. # 8 from section 3.7: Both parts are similar. We had a homework question that said that if x_n is unbounded above, then there exists a subsequence $x_{n_k} > k$ for all k . Show that this implies the subsequence x_{n_k} has limit ∞ . You can't use the squeeze theorem since we proved it was true only for finite limits (i.e., convergent sequences). Although if you can find a squeeze theorem reference for infinite limits, then it's okay. But for the exam I would expect you to address this issue—either tell me that the theorem is in the book (I don't see it right away), or prove it yourself. For this one, it's just as easy to just use the fact that you've found an $x_{n_k} > k$ and go from there as indicated above.