

## HW #3 Solutions

The Oregon Duck

October 29, 2007



*Hi kids! This is the Oregon Duck. You may all not like this, but I can't be happier that U of O beat USC this weekend in football! Go Ducks!!!! Corey and I have been close friends ever since he went to his first U of O football game when he was 12. So after such a big win for us I had way too much energy not to write the solutions for you. Next weekend: Arizona State. We can beat those guys! GO DUCKS!!!!*

- 3.4, #1. This was an example in class, and is on the exam solutions as well.
- 3.4, #5. Suppose  $x_n$  is monotone decreasing, and bounded. Let  $x = \inf\{x_n | n \in \mathbb{N}\}$ . Let  $\epsilon > 0$  be given. Then there exists a member of the set,  $x_{n_0}$  with  $x \leq x_{n_0} \leq x + \epsilon$ . Then since  $x$  is a lower bound for  $\{x_n | n \in \mathbb{N}\}$ , and since  $x_n$  is decreasing, for all  $n \geq n_0$  we have  $x - \epsilon < x \leq x_n \leq x_{n_0} \leq x + \epsilon$ . Thus the sequence  $x_n$  is eventually in the arbitrary neighborhood  $U_\epsilon(x)$ .
- 3.4, #9. We consider the supremum of  $A$ , as the other question asked has a similar solution. It suffices to consider two cases,  $\alpha \in A$  and  $\alpha \notin A$ . If  $\alpha \in A$ , then the constant sequence  $x_n = \alpha$  is a monotone sequence which converges to  $\sup A$ .
- If  $\alpha \notin A$ , and  $\epsilon > 0$  is given, then there exists an element  $x_1$  of  $A$  with  $\alpha - \epsilon \leq x_1 < \alpha$ . (Notice the inequalities, namely,  $\alpha$  isn't in  $A$  and so the  $<$  can't be  $\leq$ .) Since  $x_1 < \alpha$ , it is not an upper bound for  $A$ . So there exists an element  $x_2$  with  $x_1 \leq x_2 < \alpha$ .

Continuing in this fashion will get you a sequence  $x_n$ . Here is where Corey made a mistake (and thus didn't penalize any of you for making the same mistake). As an omnipotent and omniscient being (the Oregon Duck) I can tell these things and you're glad I'm writing these solutions and not Corey.

See, Corey concluded, as did mostly everyone else, that this sequence was monotone (which is it), and that it converges to  $\alpha$  (which it might not). See, what if  $x_n = 1 - \frac{1}{n}$  and  $\sup A = 2$ . There's no proof that this sequence converges to  $\alpha$ . So we modify the above construction somewhat to account for this lapse in mental ability. I've already called the math police and reported this incident. It will go on Corey's permanent record.

Choose  $x_1$  as above. Let  $\epsilon_2 = \min\{|\alpha - x_1|, |\alpha - \frac{1}{2}|\}$ . Then there exists an element  $x_2 \in A$  with  $x_2 \in U_{\epsilon_2}(\alpha)$ . Notice that by continuing in this fashion, we have trapped  $x_n$  by the sequence of inequalities  $\alpha - \frac{1}{n} \leq x_n < \alpha$ . Now by the Squeeze Theorem the sequence  $x_n$  converges to  $\alpha$ .

- 3.5, #2. We discussed this question at length in class. If  $x_n$  is a bounded sequence of integers, then the set  $\{x_n | n \in \mathbb{N}\}$  is finite. Thus there must be one number in the set which is repeated infinitely many times, and this is a subsequence which is constant.