



## Math 546 STOLEN Midterm of .....

*This is the clock king! I've taken a break from stealing all of Corey's free time and instead have focused on stealing his belongings! Namely, your exam! He for some reason keeps a Clock King repellent spray in his desk, and I was only able to sneak in and get this portion for you before he let me have it. So, here you go! ROCK ON!!!*

1. (25 points) Please choose exactly 3 of the following to do. No, you won't get extra credit if you do more.
  - (a) Let  $R$  be a set, and let  $+$  be a commutative binary operation on  $R$ , and let  $\cdot$  be another (possibly not commutative) operation on  $R$ . Please carefully define what it means for  $R$  to be a ..... Please also carefully state the ..... Test.
  - (b) Let  $R$  be a ring. Please carefully define what it means for  $R$  to be an ..... test.
  - (c) Let  $R$  be a ring. Please carefully define what it means for  $R$  to be a ..... Please also state the ..... test.
  - (d) Let  $\varphi : R \rightarrow R'$  be a function, and suppose  $R$  and  $R'$  are rings. Please carefully define what it means for  $\varphi$  to be a ..... Please also .....
  - (e) Let  $R$  be a ring, and let  $I$  be a nonempty subset of  $R$ . Please carefully define what it means for  $I$  to be an ..... Please also provide an example of a .....
  
2. (25 points) Please choose exactly 1 of the following to do. No, you won't get extra credit if you do more.
  - (a) Let  $R$  be a ring. .... (.....)
  - (b) Let  $I$  be an ideal of a ring  $R$ . Prove that if  $I$  is a maximal ideal, then .....
  - (c) Please prove the ..... Dream. That is, .....
  - (d) .....
  
3. (25 points) Please choose exactly 1 of the following to do. No, you won't get extra credit if you do more.
  - (a) Let  $R$  and  $R'$  be rings. Please do all of the following:
    - i. Let  $a \in R$ . Please carefully define what it means for  $a$  to be an .....
    - ii. Suppose  $\varphi : R \rightarrow R'$  is a ring homomorphism. Show that if ..... must also be .....
    - iii. Suppose  $R$  is an integral domain. ....
  - (b) Suppose  $R$  is a ring with unity 1, suppose  $R'$  is a ring, and that  $\varphi : R \rightarrow R'$  is a ring homomorphism. Please do all of the following:
    - i. Show that  $\varphi(R)$  .....
    - ii. Show that  $\varphi(1)$ .....
    - iii. Find an example of a unital ring  $R$ , and a *non-unital* ring  $R'$ , and a ....., and quietly mumble to yourself that  $R'$  doesn't have unity, but it has a subring that does.

- (c) Let  $B$  be the subring of  $M(2, \mathbb{Z})$  consisting of matrices of the form..... Show that .....
- (d) Suppose  $I$  is an ideal of a unital ring  $R$ . Please do all of the following:
  - i. ....
  - ii. .... field, then.....

4. (25 points) Please choose exactly 1 of the following to do. No, you won't get extra credit if you do more.

- (a) Please ..... and prove .....
- (b) Let  $I$  be an ideal of  $R$  that has finite index (that is, the quotient ring  $R/I$  has a finite number of elements). Please prove that ..... Please also draw a smiley-face on the upper right-hand corner of this page after ..... You may also find the next question 4(c) a sort of hint for this question.
- (c) .....
- (d) Suppose  $R$  is a ring. Please do all of the following:
  - i. ....
  - ii. ....
  - iii. ....
  - iv. ....
  - v. ....

5. For 2 free points, tell me something funny!