Math 546 Midterm Review Sheet of Awesomeness!!!

Pinky and the Brain

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Hello, future citizens of my dominated world... this is The Brain and my associate Pinky here to tell you about the upcoming exam in Math 546. Enjoy! ROCK ON!

1. Section 6.1: Examples and Basic Concepts. In this section, we learned about the concept of a ring as a special kind of abelian group. We saw examples of lots of rings, and distinguished between those which are commutative and those that contain a multiplicative unit. We learned about some basic properties (for example, $0 \cdot r = 0$ for all $r \in R$, a ring). We learned about subrings, the subring test, and applied the subring test to some examples. The basic definition of a ring, its basic properties, and the subring test (along with its application) are without a doubt the most important parts of this section.

2. Section 6.2: Integral Domains. We learned about the basic definition of an integral domain and discussed cancellation in such rings, along with a classification of the zero divisors and units in the rings $\mathbb{Z}_n$. The subdomain test is important, along with its application.

3. Section 6.3: Fields. This section mirrored the last one, in that we learned about another type of ring, a field, a learned about its associated subfield test. There is a

close relationship between fields and integral domains, and we proved that they are the same if the ring is finite. We saw that a division ring is the noncommutative version of a field, and discussed the quaternions (a great example of a division ring) in detail. Corey still owes you a typed up summary of these remarks, and hasn't forgotten. Finally, a discussion of the characteristic of a ring ensued, and we learned about its properties when applied to integral domains or, more generally, rings with identity. See Theorems 6.3.26 and Theorem 6.3.29 for details.

4. Section 7.1: Ring Homomorphisms. Here, we discussed the sorts of interesting morphisms that take us from rings to rings: these are called ring homomorphisms (to distinguish them, at least, from *group* homomorphisms). We learned how to determine, in some cases, the number of ring homomorphisms between certain rings by considering generators and by imposing the condition of it already being a group homomorphism of the underlying abelian group, and the extra condition which forces the morphism to respect the multiplicative structure in the rings. The basic properties of ring homomorphisms (Proposition 7.1.9) gave us lots of info that we have used (and will use) subsequently, and these facts are, thus, important. In particular, some of these facts have subtle requirements that are important to keep in mind. A ring isomorphism is a fundamental concept, and we showed that ring isomorphism preserved such properties of being commutative, an integral domain, or a field. We also saw an interesting application to finding certain roots of certain polynomials.

5. Section 7.2: Ideals. This section has been, by far, the longest. The main points come up, though, in the correct order. We motivate the definition of an ideal by searching for an object for which a natural multiplication on the quotient (group) R/I is defined. This gives us the definition of an ideal, and we then prove that the quotient (group) R/I is a ring with that multiplication. A discussion of the first isomorphism theorem for rings ensued, along with a discussion of how homomorphisms preserve ideals in rings (again, there were some subtle technicalities here worth looking over). We then began a discussion of prime and maximal ideals, after an appropriate motivation, and along the way learned a number of important facts (see Example 7.2.8, Proposition 7.2.11, and Theorem 7.2.27). These facts all combined to prove what is one of the centerpieces of this section, Theorem 7.2.27. My presentation of this theorem in class is perhaps more carefully and deliberately approached (than by the author) due to the importance of the result. Finally, the observation that every maximal ideal is, in fact, prime is made in Corollary 7.2.28.

6. General suggestions for studying for the exam. Generally, there will be no surprises on the exam. In fact, there will be choices for you on the exam. I'll write a bunch of questions, and you'll get to have some choice as to which questions you attempt. Of course, some will be manditory, but you'll have a bit of choice in which questions you choose to answer for the remainder of the exam. From the sections above, all

results, definitions, and examples presented in class are of interest (Corey has skipped a few results in the book, the ones in class which he has presented are what he finds "interesting"), along with any homework questions we've covered so far. The only other thing which deserves more discussion is about the material from Section 7.2, namely, how much of it will be on the exam, and to what detail. Although we can't say for sure what will be on the exam, Corey has assured me that there will be emphasis on the discussions and material we have had in class about the material from Section 7.2, and not as much attention paid to the homework from Section 7.2. Corey understands that you'll not had as much time to work on the homework problems from this section, and have not (or will not) get feedback on your work from that section in time, and therefore will not stress that material on the exam. He does, however, feel justified in asking questions related to the material presented in class, given that the exam is still 6 days away and that this material has already been discussed in class. Thus, from Section 7.2, I would expect the majority of questions to directly relate to what has been presented in class, and not mirror, as much, the homework which has been assigned from this section. That being said, it is entirely likely that some of the questions discussed in class are very similar to the homework questions assigned, and for that reason, Corey can't promise that there won't be any material from Section 7.2's homework that won't make it onto the exam. I hope that clears things up, but please do send Corey an email or stop by his office to ask any questions if it doesn't (his office hours are 12-4 on Monday). In addition, we'll have some time set aside to go over any subjects which are unclear to any of you on Monday, so study up this week and weekend, and bring with you any questions that you generate. Other than that, good luck, and ROCK ON!