HW # 2 Solutions

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Section 6.2: 1–4, 6, 8, 9, 15, 16.

1. The zero divisors are those nonzero (not multiples of 4) that share factors with 4, which is only $2 \in \mathbb{Z}_4$.

2. Same process, but these elements are $2, 4, 6$.

3. There are no zero divisors, since every positive integer less than 11 is relatively prime to 11.

4. The only zero divisors are $(0, 1)$ and $(1, 0)$.

6. The zero divisors are of the form $(n, 0)$ and $(0, r)$ for $n \in \mathbb{Z}$, and $r \in \mathbb{Q}$.

8. The only hypothesis missing to ensure such an object is an integral domain is the existence of a multiplicative identity. So the even integers $2\mathbb{Z}$ is an example.

9. The only hypothesis which is missing is that of commutativity. See the quaternions $\mathbb{H}$ on page 206 for an example. . . I'd type it all out, but we'll be discussing this in class in detail and I'll be proving it there. Now I see that I agreed to type out a whole document about the quaternions. . . stay tuned for that on the course website.

15. Notice that $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is a nonempty subset of the integral domain $\mathbb{R}$. The hypothesis of this problem assumes that $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is already a ring with the same multiplication and addition operations as $\mathbb{R}$, by definition, $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is a subring of $\mathbb{R}$. Moreover, $1 \in \mathbb{Q}(\sqrt{2}, \sqrt{3})$, so by the subdomain test, $\mathbb{Q}(\sqrt{2}, \sqrt{3})$ is an integral domain.

16. Any subdomains of $\mathbb{Z}$ must be themselves subrings, and any integral domain must contain the multiplicative identity, in this case, the integer 1. So any subdomain must contain repeated additions of 1, that is, $n \cdot 1 = n$ for any nonnegative integer n. Moreover, the additive inverse of 1 must also be in any ring which contains 1. So any subdomain of $\mathbb{Z}$ must contain -1 as well. But then such a subdomain must also contain all of $n \cdot (-1) = -n$ for any nonnegative integer n. So any subdomain must contain all positive, and negative integers, in addition to containing 0. So the only subdomain of $\mathbb{Z}$ is $\mathbb{Z}$ itself.