

Math 546 Final Exam Review

My Math 301A class

June 15, 2009

Hello, from Corey's Math 301A course! We hear all about what sorts of stuff Corey discusses in your Math 546 course, and we decided it would be fun to type out a review sheet for the final. Of course, Corey has already posted the infamous "Stolen Exam", and he suggests that this should augment your studying efforts. Good luck, and ROCK ON!

1. Section 8.1: Basic Concepts. In this section we learned a lot about the basic definitions of polynomials, their sums and their products. We also re-introduced ourselves to familiar verbage such as "leading coefficient," "degree," etc. Theorem 8.1.12 assures us that polynomial rings are well-behaved. In particular, part (4) of that theorem has helped us on a number of occasions. That D : integral domain $\Rightarrow D[x]$: integral domain has also been of use, as has the understanding of the units of $R[x]$, see Proposition 8.1.14 and Corollary 8.1.15. Finally, we have used (on few occasions, that $Char(R) = Char(R[x])$. The homework has a few questions on the ring of formal power series $F[[x]]$ which were of interest as well.
2. Section 8.2: The Division Algorithm in $F[x]$. As advertised, this section contained a statement of the division algorithm, and Corey presented it as an analogy to the division algorithm over \mathbb{Z} . This gave rise to another important definition, the gcd of two polynomials, and the ever-so-important Euclidean Algorithm. We will see later how important it is to be able to express $gcd(f, g) = f \cdot u + g \cdot v$.
3. Section 8.3: More applications of the division algorithm. The Factor Theorem and the Remainder theorem function as wonderful applications of the division algorithm. In addition, the number of zeros of a polynomial (of finite degree) being finite (equal to the degree of the polynomial) is an extremely important fact. Example 8.3.8 shows why F being a field is important in that respect, and finally Theorem 8.3.12 about complex roots of polynomials with real coefficients was of interest as well.
4. Section 8.4: Irreducible Polynomials. The notion of an irreducible polynomial plays a crucial role in this course, but namely in this section, it's role in the unique factorization theorem 8.4.6. In addition several results about the possible irreducibility of polynomials (Theorems 8.4.7, 8.4.11, 8.1.6, 8.4.19, , and 8.4.24) proved to be useful.

You may find it useful to study 8.4.22 as an application of Eisenstein's criterion that we didn't go over in class.

5. Section 8.6: Ideals in $F[x]$. This section pointedly asserted that $F[x]$ is a PID, and that as a result, prime and maximal ideals coincided with principle ideals generated by irreducible polynomials (but not necessarily over a commutative ring R which is *not* a PID, for example, see Example 8.6.11.)
6. Section 8.7: Quotient Rings of $F[x]$. This final section was devoted to understanding the algebra one does in the field $F[x]/\langle p(x) \rangle$ (where $p(x)$ is irreducible). Proposition 8.7.5 really sums up our interest in this section, and encourages you to try proving it yourself—even though Corey really recalled the main points on the last day of class.
7. General Suggestions for studying for the final: I suggest that you study for the final. Really, this review sheet and the “Stolen Final” suggest what topics are important, and what you can expect when you sit down to take the exam on Wednesday, June 17th at 6 PM. You'll have lots of time, and I suggest you read all the questions and carefully choose which ones you want to try. And, of course, should you have any questions before the exam, feel free to email Corey or stop by his office, and he'll do his best to help you out. Other than that, good luck, and ROCK ON!