

HW # 7 Solutions

Pacman, after being arrested

March 18, 2008



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1. Number 2: There are 6 cosets of H in G , as $|S_4 : H| = \frac{|S_4|}{|H|} = \frac{24}{4} = 6$.
2. Number 16: We use Fermat's little theorem. Let a be relatively prime to n . Then $a \in U(n)$, and thus $a^{|U(n)|} = a^{\varphi(n)} = 1 \pmod n$.
3. Number 20: Let $H, K \leq G$. Then note that $H \cap K \leq H$ and $H \cap K \leq K$ as well. And the order $H \cap K$ must divide both H and K . But the only common divisor of $|H|$ and $|K|$ is 1, so $|H \cap K| = 1$.
4. Number 43: Notice that $aH = bH$ if and only if $b^{-1}a \in H$. So if a and b are matrices in $Gl(n, \mathbb{R})$, then $b^{-1}a \in H$ means $\det(b^{-1}a) = \pm 1$. But then $\pm 1 = \det(b^{-1}a) = \det(b)^{-1}\det(a)$. So $\det(b) = \pm \det(a)$. All of these arguments are reversible as well, so the converse is true.