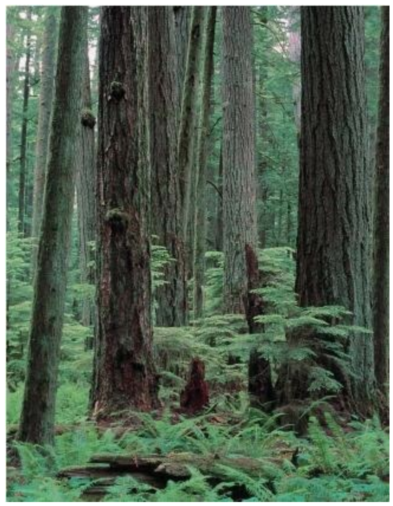
Solutions to HW # 5

The Skog

March 11, 2008



Hi everyone. Skog is the Swedish word for forest, and I'm comin' at'cha with the solutions to the graded problems from Chapter 5. KUMON!

1. Number 3: (a) $|(124)(357)| = lcm(3,3) = 3$, (b) $|(124)(3567)| = lcm(3,4) = 12$, (c) $|(124)(35)| = lcm(3,2) = 6$, (d) $|(124)(357869)| = lcm(3,6) = 6$, (e) Notice that $(1235)(24567) = (124)(3567)$, so $|(1235)(2467)| = lcm(3,4) = 12$, (f) Notice that $(345)(245) = (25)(43)$, so $|(345)(245)| = lcm(2,2) = 2$.

2. Number 6: We must construct an element of order 15 in A_8. So in particular, the element must be even. So set $\sigma = (123)(45678)$. This is the product of even permutations, and is thus even. $|\sigma| = lcm(3,5) = 15$, since the cycles that make up σ are disjoint.

3. Number 42: We first show that β and β^{-1} have the same parity. Suppose that $\beta = \tau_1 \cdots \tau_s$, where the τ_i are transpositions. Then by the socks and shoes principle, $\beta^{-1} = \tau_s^{-1} \cdots \tau_1^{-1}$, and since each of the transpositions are of order 2, $\tau_i^{-1} = \tau_i$. Then

$\beta^{-1} = \tau_s^{-1} \cdots \tau_1^{-1} = \tau_s \cdots \tau_1$, and so since β and β^{-1} can be expressed as the same number of transpositions, it follows that β and β^{-1} have the same parity.

Now consider the product $\beta\alpha\beta^{-1}$, and suppose that β and β^{-1} can be expressed as the product of s transpositions. Suppose also that $\alpha = \sigma_1 \cdots \sigma_r$, where the σ_i are transpositions. Then $\beta\alpha\beta^{-1}$ is a product of $s + r + s = 2s + r$ transpositions. Since r and $2s + r$ have the same parity, α and $\beta\alpha\beta^{-1}$ have the same parity.

4. Number 51: Suppose σ is the product of r transpositions. Then σ^k is the product of rk transpositions. We must show that if the order , k, of σ is odd, then r is even. So if $\sigma^k = e$, then σ^k is even, and so rk must be even. Since k is odd, it follows that r is even.