

# Math 545 final exam review sheet!

This Carl tee-shirt

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1. Chapter 5: Permutation groups. This chapter was very important. We learned about the symmetric groups  $S_n$ , the group of bijections of the set  $\{1, 2, \dots, n\}$  to itself under composition. We learned that we could represent each element of  $S_n$  in disjoint cycle form, and how to compute the order of such an element in such a form as the least common divisor of each of the cycle lengths. Moreover, disjoint cycles commute. After that we showed that each permutation can be written (not uniquely!) as a product of transpositions, and that the notion of parity is well-defined: each element cannot be written as the product of an even number of transpositions and an odd number of transpositions. This, as a consequence of the identity being an even permutation, whose proof is short and nontrivial. This led us to the definition of the alternating group as the subgroup of all even permutations, having index 2 in  $S_n$ .
2. Chapter 6: Isomorphisms. This is also a very important chapter, as the notion of isomorphism was introduced. Every group is isomorphic to a group of permutations (Cayley's theorem). We also learned about homomorphisms in general at this stage, and Corey encourages you to compare Theorem 6.2 and Theorem 6.3 with Theorem 10.1 and 10.2. Many of the facts are the same, some are slightly different, and all of them are important and too numerous to list here. We also learned about inner automorphisms and automorphisms in general, and that  $\text{Aut}(\mathbb{Z}_n) \cong U(n)$ , a basic and first result of that sort.

3. Chapter 7: Cosets and Lagrange's Theorem. This contains a theorem of finite group theory that is absolutely fundamental: Lagrange's theorem. To understand the proof of the theorem, one needs most of the properties of cosets in the Lemma on page 138. Several corollaries (pages 141 and 142) also illuminate the importance of Lagrange's theorem. Moreover, we were able to classify all groups of order  $2p$ , where  $p$  is an odd prime. Finally, the orbit-stabilizer theorem was a fun application of cosets to counting.
4. Chapter 8: External direct products. The external direct product is defined, and the order of elements in the direct product group is studied. In addition, a description of when  $G \oplus H$  is cyclic when  $G$  and  $H$  are cyclic is helpful when trying to understand abelian groups, and this proved to be helpful in many ways when determining the isomorphism class of finite abelian groups! We also studied the  $U(n)$  groups, and saw that these decompose as direct sums,  $U(st) \cong U(s) \oplus U(t)$  when  $\gcd(s, t) = 1$ .
5. Chapter 9: Normal subgroups and factor groups. We learned the definition of a normal subgroup, and learned several ways to check to see if a subgroup is normal. For instance, the normal subgroup test (page 178), but also direct manual checking that  $aH = Ha$  for all  $a \in G$ . This, of course, isn't as cumbersome as it sounds until one realizes you need only check this relationship on elements  $a \in G$  which represent different coset representatives. Afterwards, we introduced the notion of a factor group, and developed several useful applications of this algebraic object. See pages 184–186 for a few of these applications. We skipped internal direct products for another day, and then only had a brief discussion about these objects that will not be on the final exam.
6. Chapter 10: Group Homomorphisms. We covered much of this section in Chapter 6. Corey draws your attention to Theorems 10.1 and 10.2, and to the corollary on page 203. In addition, he emphatically draws your attention to the first isomorphism theorem on page 206 (all of this in your notes, of course, as well). Corey suggests looking at problems 7–9, 14, 18, 19, 28, 35, 40, 50, 53, 54, 57. Of course, these are meant for practice only and shouldn't be turned in. They just exemplify some of the concepts this chapter has to offer, and would be wonderful practice for you all as you study for the final. Stay tuned for a garbled up version of the final! Good luck, and

**ROCK ON!**