

## Math 545 stolen exam!



*Hi, I'm a Pluton, that annoying character from the old skool NES game "Kid Icarus" that would steal all of your weapons, and take them to the black market slumlord who would sell you your own hot merchandise at an exorbitant cost. If you don't know what I'm talking about, then here's the deal: I've stolen your test. I came late at night to Corey's office thinking he wouldn't be here, but I was wrong. For some reason, Corey seems to have work to do that keeps him at the office quite late most nights. Well anyway, I managed to escape, but only after Corey ripped important information from the exam away. So I list what is left of the tattered exam. The dots ..... indicate that words were once there, but I can no longer make out what they were. Sorry! I hope it helps! ROCK ON!!!*

1. (20 points) Please choose any two of the following to do.
  - (a) Let  $G$  be a set, and let  $\cdot$  be a binary operation. Please carefully define .....
  - (b) Let  $G$  be a group, and let  $H \subseteq G$ . Please carefully define .....
  - (c) Please carefully state any subgroup test.
  - (d) Let  $a \in G$ , a group. Please carefully define .....
  
2. (20 points) Please choose exactly one of the following to do.
  - (a) Please do both of the following parts:
    - i. Please define the group  $U(12)$ , list ..... Cayley table for .....
    - ii. Is  $U(12)$  .....? Why or why not?
  - (b) Please do both of the following parts:
    - i. In the dihedral group  $D_3$ , let  $R$  be rotation clockwise by  $120^\circ$ , and let  $F$  be a reflection along the vertical axis. The set  $D_3 = \{e, F, R, R^2, RF, R^2F\}$ , and satisfies the relation  $FR = R^{-1}F$ . Please .....
    - ii. Please compute .....
  - (c) Please do both of the following parts:
    - i. Please carefully define the group..... Be sure to mention the group operation.
    - ii. Please show that the subset ..... is a subgroup of  $GL(n, \mathbb{R})$ .
  
3. (20 points) Please choose exactly one of the following to do.
  - (a) For each of the items below, there is a set  $H$  which is a subset of a given group  $G$ . Please either prove that they are subgroups, or list at least one group axiom that fails.
    - i. ....
    - ii. ....
    - iii.  $\{a + b\sqrt{-1} \mid a^2 + b^2 = 1\} \subseteq \mathbb{C}^* = \{\text{the nonzero complex numbers under multiplication}\}$ .
  - (b) Prove that  $\mathbb{Q}^* = \{\text{the nonzero rational numbers under multiplication}\}$  is not .....
  - (c) Give an example of a group  $G$  and elements  $a, b \in G$  so that  $a$  has finite order, and  $b$  .....
  
4. (20 points) Please choose exactly one of the following to do.

- (a) .....
- (b) Let  $G = \{f : \mathbb{R} \rightarrow \mathbb{R} \mid f \text{ is invertible}\}$ . Let the operation on this set be function composition. The set  $G$  with the given operation is a group, and this fact need not be proven for this problem.
  - i. ....
  - ii. Let  $f(x) = 2x + 1 \in G$ . Please list one linear function  $g(x) = ax + b$  where ....., but  $g(x) \notin Z(G)$ .
- (c) Please find ..... Please also show your work.

5. (20 points) Please choose exactly one of the following to do.

- (a) We know that  $\mathbb{Z} = \langle 1 \rangle$  is a cyclic group. Please show that there are no .....
- (b) Let  $a \in G$ , a group. ....
- (c) Notice that  $128 = 2^7$ . Please do all of the following parts.
  - i. ....
  - ii. ....
  - iii. What are the generators of each of these subgroups, and .....