Name: ______________________________

Math 545 Final Exam of Sooper Awesomeness! Stolen by the Hamburgler!!!!

Okay kids! Here's what I could pry from Corey's hands before running down the hallway! I'm off to hold up another McDonald's! Sorry it's not everything, but it's all I could get! Good luck, and Rock on!

1. (25 points) .... *Gee, from what I could see, this question looks a whole lot like the first question of the midterm!*

2. (25 points) Please choose any two of the following to do.

   (a) Let $H \leq G$. Please carefully define what it means for H to be a ....

   (b) Suppose $H \trianglelefteq G$. Please carefully define the group .... Be sure to include a description of the binary operation.

   (c) .... isomorphism ....

   (d) Please carefully state .... Theorem.

   (e) Please carefully state .... for abelian groups.

3. (25 points) Please choose exactly one of the following to do.

   (a) Please do all of the following. Let $\sigma =$ ....

      i. Please express σ in disjoint cycle notation.

      ii. Please compute $|\sigma|$.

      iii. Please compute ....

      iv. ....

      v. Please find a nonidentity element $\tau \neq \sigma$ so that ....

   (b) Please do all of the following.

      i. Please carefully define the group $U(12)$. ....

      ii. Is $U(12)$ ....? Why or why not?

      iii. Please show that $U(12)$ is isomorphic to ....

   (c) Please do all of the following.

      i. ....

      ii. Find two reasons why ....

      iii. Why must $< 12 > \cap < 18 >$ be cyclic? Find a ....

4. (25 points) Please choose exactly one of the following to do.

   (a) Please do all of the following parts. Let $H = \{e, (12)\} \leq S_3$.

      i. Please list the......................... in S_3.

   ii. Is ....

(b) Please do all of the following parts. Let $H = \{A \in Gl(n, \mathbb{R})|$.....................

   i. Show that $H \leq Gl(n, \mathbb{R})$.

   ii. Show that ....

(c) Please do both of the following parts. Let $Z(G)$ denote the center of G.

   i. Prove that ....

   ii. Prove that if G is abelian, then .... normal.

5. (25 points) Please choose exactly one of the following to do.

   (a) Please do both of the following parts.

      i. Prove directly (without citing any results) that ....

      ii. Is ....? Why or why not?

   (b) Please do both of the following parts.

      i. Let φ be an isomorphism of groups. Show that ....

      ii. Either prove that ...., or prove that A_4 is not isomorphic ....

   (c) Please do both of the following parts.

      i. Prove that if $gcd(a, n) = 1$, then ....

      ii. What is the last digit of the number ....?

6. (25 points) Please choose exactly one of the following to do.

   (a) Please do both of the following.

      i. Let $\beta \in S_n$. Prove that β and β^{-1} have ....

      ii. Prove that for any $\alpha \in S_n$ that ....

   (b) Let $\varphi : G \to H$ be a homomorphism. ....

   (c) Prove that any group of .... order ..... is ....

7. (25 points) Please choose exactly one of the following to do.

   (a) Prove that $Inn(G) \trianglelefteq$............................

   (b) Note that $|A_4| = 12$, and that $6|12$. Prove that A_4 has no ....

   (c) Please prove that if $G/Z(G)$ is ...., then G is ....

   (d) Please do both of the following parts. Let G be an abelian group.

      i. Suppose p is a prime number, and that p divides the order of a finite group G. Show that there are at least $p - 1$ distinct elements of order p.

      ii. Give an example that shows..........................

8. (25 points) Please choose exactly one of the following to do.

   (a) Please state and prove the 1st isomorphism theorem.

   (b) Classify, up to isomorphism, all groups of order k, for .... You may assume that every group of order p^2, for p a prime, is abelian.

   (c) Please do both of the following parts. Suppose G is an abelian group. Define $G^p = \{g^p | g \in G\}$, the group of all elements of G raised to the pth power.

      i. Show $G^p \leq G$.

      ii. Suppose that p is a prime that divides the order of the group. Show that ....

   (d) Please prove the .... isomorphism theorem: Suppose .... Prove that

   blah blah blah.

   (that is, ... Very cute.)

9. For 3 free points, tell me ....!