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The Carebears

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15. Show that if G is a group, then $Z(G) = \cap_{a \in G} C(a)$.

Proof. We show that $Z(G) \subseteq \cap_{a \in G} C(a)$ by showing that $Z(G) \subseteq C(a)$ for every a in G. Then we show that $\cap_{a \in G} C(a) \subseteq Z(G)$.

Let $x \in Z(G)$. We must check that, for any a in G, we have $xa = ax$. But since $x \in Z(G)$ and $a \in G$, by definition, $xa = ax$.

Let $x \in \cap_{a \in G} C(a)$. Then $x \in C(a)$ for every $a \in G$. Let $y \in G$. We must show that $xy = yx$. But since x is in every centralizer $C(a)$ for any $a \in G$, $x \in C(y)$ since $y \in G$. So by definition, $xy = yx$.