Name: ______________________________

STOLEN Math 531 Exam # 1 of Awesomeness!!!

Hi everyone, Robin Hood here! I was walking down Jack Brown hall during my yearly visit of each Cal State (delivering the money I stole from the rich to give to the CSU). I saw Corey working on the exam and thought I could steal it from him. But, instead of going quietly, Corey because agitated and refused to let it go. I managed to rip the following out of his hands, and I pass it along to you. Good luck, I hope it helps!

Directions: Please take this exam without cheating. It's worth 100 points. Please also read the directions to each question carefully–questions that you do not follow the directions on may result in a score of zero points for that question. You may use a writing utensil, a calculator, and your brain. Oh, and ROCK ON!!! Unless otherwise informed, for this exam, V will always denote a finite dimensional vector space, $T : V \to V$ will be a linear operator, and A will be a square matrix (of an appropriate size). In addition, if β is a basis for V, you don't need to explain the notation $[T]_\beta$ as the matrix representation of T with respect to β (unless you choose to use a different notation).

1. (25 points) Please do exactly two of the following.
   (a) Please carefully define what .... Please also carefully define what ....
   (b) Please carefully define what it means for T to be .... Please also define what it means for a matrix ....
   (c) Please carefully define the......................... Please also prove that if W is a T-invariant subspace and T_W is the restriction of T to W, then ....
   (d) Let λ be an eigenvalue of T. Please carefully define the *eigenspace* E_λ. Please also prove that ....

2. (25 points) Please do exactly one of the following.
   (a) For this problem, let A be the matrix given below. Please do all of the questions that follow.

   $$A = \text{........................}$$

      i. Please find all eigenvalues of A, and ....
      ii. For each eigenvalue λ of A, please determine $\dim(E_\lambda)$.
      iii. Is ....? If not, please explain why. If so, please ....
   (b) Suppose .... eigenvalues of T with corresponding .... $v_1, \ldots, v_k$. Please prove that $\{v_1, \ldots, v_k\}$ is linearly independent.

(c) Suppose λ is an eigenvalue of T. A *Jordan block* of size k is a $k \times k$ matrix of the form:

$$\mathcal{J}(\lambda, k) = \begin{bmatrix} \lambda & 1 & 0 & 0 & \cdots & 0 \\ 0 & \lambda & 1 & 0 & \cdots & 0 \\ 0 & 0 & \lambda & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & \lambda \end{bmatrix}.$$

Prove that if $T = \mathcal{J}(\lambda, k)$ on a vector space V of dimension ....

3. (25 points) Please do exactly one of the following.

   (a) Suppose T has n distinct eigenvalues. Prove that .... Is the converse true? If yes, prove your claim. If not, then please provide a counterexample and give a detailed reason as to why it is a counterexample.

   (b) A square matrix .... $A^k = 0$, the zero matrix. Please do all of the following:

      i. ...., then 0 is the only eigenvalue of A (if there are any eigenvalues of A).

      ii. ....

   (c) Suppose the only eigenvalue of T is λ, and that T is diagonalizable. Prove ....

4. (25 points) Please do exactly one of the following.

   (a) Suppose that λ is an eigenvalue of T, and that the multiplicity of λ is m. ....

   (b) For this problem, let A be as below. Please do all of the parts that follow.

   $$A = \text{........................}$$

      i. Please compute the characteristic polynomial of A.

      ii. Please form W, the T-cyclic subspace generated by the first basis vector e_1. What is a basis for W?

      iii. ....

      iv. .... smile excitedly ....

   (c) .... the matrix A^k, where A is the 2×2 matrix given below.

   $$A = \text{........................}$$

5. For 2 free points, tell me something funny!