Math 531 Awesome Exam Review Sheet

March Madness!!!

March 17, 2010

Good evening!!! Corey is hard at work looking at his March Madness picks, so I thought I'd write you this exam review sheet. Enjoy! Keep your eyes peeled here on the website for the stolen exam!

1. Section 5.4: Invariant subspaces and the Cayley-Hamilton Theorem. We covered this section on the exam, with exception to the stuff about direct sums of matrices, and how the direct sum of (invariant) subspaces impacted what we knew about linear operators on a space. See Theorem 5.24 and 5.25 for reference as to the important facts from this section not necessarily already covered on the midterm review sheet.

2. Section 6.1: Inner products and norms. We learned about the definition of an inner product space, and various basic properties of them. The adjoint of a matrix is discussed, and the notion of what it means for vectors to be "orthogonal" or "orthonormal" is also introduced.

3. Section 6.2: Gram-Schmidt and Orthogonal Compliments. We learned various basic facts leading up to Theorem 6.4, which is essentially the "Gram-Schimdt" process, which inputs a basis and spits out an orthonormal basis. Much can now be done that we know that orthonormal bases exist. We also learned about $\perp$ spaces, and that if W is a subspace of V, and $\dim(V) < \infty$, then $V = W \oplus W^{\perp}$, and that orthogonal projection $\pi_W : V \to W$ was described.

4. Section 6.3: The Adjoint of a linear operator. We proved an interesting result about linear functionals (linear maps from V to the ground field F) on a finite dimensional inner product space (see Theorem 6.8), and this led us to the fact that any linear operator on a finite dimensional vector space has an adjoint (see Theorem 6.9). One big difference between this and other sections (for instance, from Chapter 5), is that our first big result required a discussion of an *orthonormal* basis. In Theorem 6.10, we relate the matrix representation of an operator with its adjoint with respect to an orthonormal basis., and that has paid huge dividends, and is an important result.

5. Section 6.4: Normal and Self-Adjoint Operators. We discuss normal and self-adjoint operators, and begin this study in earnest with Schur's Theorem, on page 370. Then

go on to describe normal operators. Using Schur's Theorem, we prove that on a complex inner product space, normality is equivalent to orthogonal diagonalizability (that is, there exists an orthonormal basis of eigenvectors). Then, we prove that on a real inner product space, that self-adjointness is equivalent to orthogonal diagonalizability.

6. Section 6.5: Unitary and Orthogonal operators and their matrices. Various properties of these operators are discussed, and so is *orthogonal* and *unitary equivalence*. A stronger version of orthogonal diagonalizability is found in Corollary 1 and 2, on pages 381 and 382.

7. Section 6.6: Orthogonal projections and the Spectral Theorem. We discussed some prerequisite information regarding orthogonal projections (in particular, about the difference between projections and *orthogonal* projections). Then, we proved one of our main results of the course, the Spectral Theorem, page 401. Given all of the preliminary work we've done, the proof of that theorem was almost a formality. The corollaries we discussed in class are of good use as well.

8. Section 7.1: Jordan Canonical Form I. This section we proved two main results. First, that $V = \oplus K_\lambda$, the direct sum of generalized eigenspaces, and next, that there exists a basis for V made up of disjoint cycles of generalized eigenvectors–all of this only when the characteristic polynomial splits. We will discuss various aspects of *finding* such a basis in practice in class on Monday, but the theoretical aspects of these results (and the prerequisite results leading up to them) are of great use.

9. General suggestions. I suggest you study for this exam, and that you take a long look at all of the assigned homework questions and solutions that Corey has posted. Also, since the final exam is cumulative, it would be a good idea to review everything from that exam as well. Generally, you can expect an exam very similar to the midterm: a question mainly about definitions, and then some other questions where you have some amount of choice. Good luck, and Rock on!!!!