

Math 355 Quiz # 2 Solutions

By: The time 6:15 PM

June 4, 2008

It's about 6:15 PM and I'm here in Corey's office. I'm giving him some pity since he really wants to go home, but is still here working. Working. Working. Busy day. But I'm happy when he's here, it makes the office just a little less boring, and so I thought I'd repay him today by writing his solutions for him to the quiz. Hope you enjoy them!

1. Let $\varepsilon > 0$ be given. Choose N so that $\frac{15}{N} < \varepsilon$. Then for any $n \geq N$, we have

$$\left| \frac{3n}{n+5} - 3 \right| = \left| \frac{-15}{n+5} \right| \leq \frac{15}{n} \leq \frac{15}{N} < \varepsilon.$$

2. (a) We rearrange the sequence in an effort to apply properties of sequences to help us determine what value the sequence converges to. Notice that

$$\frac{2n+1}{n^2-8} = \frac{n^2 \frac{2}{n} + \frac{1}{n^2}}{n^2 \left(1 - \frac{8}{n^2}\right)} = \frac{\frac{2}{n} + \frac{1}{n^2}}{1 - \frac{8}{n^2}} \rightarrow \frac{0}{1} = 0.$$

(b) Notice that $|(-1)^n \frac{1}{n}| = \frac{1}{n}$, and that $\frac{1}{n} \rightarrow 0$. Thus, by a result in the book, $(-1)^n \frac{1}{n} \rightarrow 0$ as well.

(c) (This part is a more general statement to part (b)) Notice that $|(-1)^n y_n| = |y_n|$. Since $y_n \rightarrow 0$, we know that $|y_n| \rightarrow 0$. So $|(-1)^n y_n| \rightarrow 0$, and by the same result referenced in part (b), it follows that $(-1)^n y_n \rightarrow 0$ as well.