

Math 355 Midterm review sheet of coolness!

Bugs Bunny and his new love interest

May 2, 2008



EHHHHHHHHHHHHHHHH, what's up, doc?! My new girl and I have been sitting in on Corey's class and have some thoughtful advice concerning the midterm coming up. This review sheet, of course, isn't the most complete collection of facts regarding the class, but, as they say, it's close enough for government work. We hope it helps! And, ROCK ON!

1. Section 2.1: Algebraic and order properties of \mathbb{R} . This section really didn't have much that we didn't already know: the properties of the real numbers that we grew up with, the notion of absolute value, and the triangle inequality. Really, though, they have a place in this course, as we'll be needing these things later—especially the triangle inequality. Even the proof of the triangle inequality I wouldn't worry so much about, but number 11 from this section, which DOES use the triangle inequality, is worth studying.
2. Section 2.2: The completeness axiom. We all received our license to use the symbol " ∞ " in this section, and understand it not as a real number, but being a member of the *extended real number system*, $\mathbb{R}^\#$, a place where we may not always add/subtract/multiply/divide (the infinities $\pm\infty$), but sometimes we can, and that these algebraic operations will 1) be more useful later, and b) when they are more useful, really, they're just a matter of convenience. All

the same, after defining the supremum and infimum of a subset of \mathbb{R}^\sharp , the crowning jewel from this section is the completeness axiom, which stated that any set which is bounded above has a supremum which is real. This seemingly self-evident fact follows from a basic construction of the real numbers, which Corey has wisely omitted from this class in the interests of keeping his sanity (and yours), but may return to it later. The homework from this section is great practice, as are all of the examples Corey did in class.

3. Section 2.3: The rational numbers are dense in the real numbers. This section pretty much contains what it advertises, a proof that the rationals are dense in the reals. We also, however, discuss an important preliminary that stands on its own as a useful mathematical fact: The Archimedean principle. There are many great questions to ask from this section, although Corey wouldn't burden you with the proof of either the Archimedean principle or the denseness of the rationals in the reals. But that doesn't mean that he wouldn't ask other related questions that are related to that whose difficulty is more tolerable!
4. Section 2.4: Cardinality. We discussed the meaning behind the statement "there are just as many even integers as there are integers". Of course, we agreed that this sounded absurd, but in this section we developed the tools to describe the best generalization of the "cardinality" of an infinite set, and this was one of the consequences. We had a very specific meaning for two sets to have the same "cardinal number", and Corey suggests that you commit this to memory, and develop a facility for its use. There are lots of good questions to ask, and Corey suggests that these questions could come from this section's homework, this section's examples or class lecture, or even homework from previous sections (see 9 and 10 from Section 1.3).
5. Other comments about the exam: Corey will certainly have one question asking you to define things. Corey is actually pretty anal about this (just ask any of his former Math 553 students), and does not award much partial credit for answers which are not an equivalent formulation of exactly what Corey is asking about. For example, if Corey asked "Please carefully define a sequence in a set X ," bad answers would include "It's a bunch of real numbers", or "It's the a_n which are different in X ", or "It's the stuff about the things and junk." I hope you can see why these are either incorrect (X may not be a set of real numbers, for instance), inaccurate or incomplete (in the second bad answer, what is a_n , and why must they be different?), or, well, just a waste of Corey's time (the third bad answer). The correct answer may be found on page 33 (Definition 2.11), and only logically equivalent answers would be given full credit (or, really, any credit, except for rare circumstances). Topics that are fair game include the sup and inf of a set, what a sequence is, the definition of the (direct) image $f(A)$ where $f : X \rightarrow Y$, and $A \subseteq X$, the completeness axiom, the Archimedean property, the triangle inequality, what a *dense* subset of the real numbers is, and what it means for sets A and B to have the same cardinal number (i.e., what $A \sim B$ means in that context). This list isn't necessarily complete, but really, these definitions are extremely important for the rest of the class (and your mathematical lives in general) and shouldn't be overlooked. Other than that, a format like the quiz can be expected. Good luck, and ROCK ON!