Quiz #3 Solutions

Corey's Malfunctioning Internet Connection

March 17, 2007

Hello, future victims of internet malfunction! I recently have infested Corey's internet service provider at home and refuse to let Corey access the internet, forcing him either into the office or friendly coffee shops that have more reliable connections. It's only a matter of time before chaos and the apocalypse is upon us!

1. Let $u = (2,5)$, and $v = (-5,2)$.

   (a) Using the standard dot product as our inner product, we have $\langle (2,5), (-5,2) \rangle = 2(-5) + 5(2) = -10 + 10 = 0$.

   (b) By definition, yes, they are orthogonal.

   (c) With the new inner product, $< (2,5), (-5,2) >= 2(2)(-5) + 3(5)(2) = -20 + 30 = 10$.

   (d) No, since their inner product is different than zero, these are not orthogonal with respect to the inner product $< \cdot, \cdot >$.

(e) The number we are looking for is $\cos\theta = \frac{<u,v>}{||u||||v||}$. The numerator we computed in part (c). Recall that the length $||u||$ of a vector u is measured by the inner product in question. So, we use $||u|| = \sqrt{< u, u >}$, with the inner product given in part (c). So:

$$||u|| = \sqrt{< u, u >} = \sqrt{2(2)(2) + 3(5)(5)} = \sqrt{83},$$

and for v we have

$$||v|| = \sqrt{< v, v >} = \sqrt{2(-5)(-5) + 3(2)(2)} = \sqrt{62}\,.$$

2. We let $\tilde{w}_1 = v_1 = (1, 0)$. Then set $\tilde{w}_2 = v_2 - c\tilde{w}_1 = (1, 1) - c(1, 0) = (1 - c, 1)$. We want $\langle \tilde{w}_1, \tilde{w}_2 \rangle = 0$. So we find c so that it is:

$$\langle \tilde{w}_1, \tilde{w}_2 \rangle = \langle (1, 0), (1 - c, 1) \rangle = (1)(1 - c) + (0)(1) = 1 - c\,.$$

So we set $c = 1$, and $\tilde{w}_2 = (0, 1)$. One can check that these vectors are already of unit length, so $w_1 = \tilde{w}_1$, and $w_2 = \tilde{w}_2$. This is an orthonormal basis for $\mathbb{R}^2$.

Rock on!!!