Quiz #1 Solutions

By: Yogi Bear

Hi kids! I'm straight from the forest to give you the solutions to the last quiz you took. That, and of course, the super bowl is next weekend, and I thought I'd come out to the game and scare the pants off of some people–I mean, I look pretty harmless in the picture above, but I'm a bear! I eat people! ROCK ON!

1. You're asked to solve the linear system $3x_1 + x_2 = 6$, and $x_1 + 3x_2 = 10$. One could do this by several methods, but I'll simply subtract 3 times the second equation from the first, getting: $-8x_2 = -24$, or $x_2 = 3$. Plugging this into the first equation gives us $x_1 = 1$.

2. Most everyone did very well on this problem, although some of you screwed up some signs. So rather than typing out all of the steps, I'll just list the answers for reference:
   (a) $\begin{bmatrix} 9 & 13 \\ 5 & 1 \end{bmatrix}$
   (b) Can't add these matrices.
   (c) $\begin{bmatrix} -29 & -1 \\ -17 & -21 \end{bmatrix}$
   (d) $\begin{bmatrix} 42 & -32 & 38 \\ -9 & 14 & -23 \end{bmatrix}$
   (e) Can't multiply these
   (f) $\begin{bmatrix} 46 & 34 \\ -1 & -1 \end{bmatrix}$

(g) and (h) The answer to both of these is -12. I didn't need you to actually do all three of $\det(AC), \det(A)$, and $\det(C)$. But the fact that $\det(AC) = \det(A)\det(C)$ is useful. If you would have pointed out this fact, and referenced your result from the previous problem that would have been fine (in fact, preferable!)

3. This IS a vector space. Many of you were able to give three reasons, but many of you were very vague in your reasoning. I'll have some more comments about this at the end of this document, so look there too.

4. This IS NOT a vector space. Almost every axiom fails, but some fail more spectacularly than others. For example, this set is not closed under addition: $\vec{u} + \vec{v} \notin S^2$ if $\vec{u}$ and $\vec{v}$ are in S^2. So that's a pretty spectacular failure. As a more subtle failure, the commutativity of addition axiom doesn't really work. Sure, as vectors in $\mathbb{R}^3$ we would have $\vec{u} + \vec{v} = \vec{v} + \vec{u}$, but since the vector $\vec{u} + \vec{v}$ isn't actually IN S^2, we would have a hard time measuring if $\vec{u}+\vec{v} = \vec{v}+\vec{u}$; keep in mind if S^2 is our set we can't just say it would work out IF... it either does or it doesn't. So now you see that some of the axioms are necessary for the others to make sense. Another example is that there is no zero vector, since the only vector that works would be $\vec{0} = (0, 0, 0) \in \mathbb{R}^3$. But this vector is not in S^2. So it's arguable whether or not there exist additive inverses. Sure, I give you a $\vec{u}$ and you produce what you think the additive inverse should be $-\vec{u}$, but it doesn't enjoy the desired property, namely, $\vec{u} + (-\vec{u}) =$??? It equals the zero vector, but if our playground is only those points in S^2 then I guess we can't really say that it does what it should. See also below.

5. General Comments. Everyone did very well, the average was in the mid 40's out of 50. The only thing that really disturbed me was that people were very vague with their reasonings in questions 3 and 4. The question asks you to state one or more properties, and there were some answers that didn't look like the axioms of a vector space. Also, I saw some people multiplying things together in question 3... yes, you can multiply functions, but the axioms of a vector space say that you only need to show properties of addition, and *scalar* multiplication. In question 4 I saw somewhat sloppy notation. Let me give you an example of what would be a great answer:

   "Suppose $(x, y, z) \in S^2$. Then $x^2 + y^2 + z^2 = 1$. If $c \in \mathbb{R}$, and $c \neq 1$, then I can multiply $c^2x^2 + c^2y^2 + c^2z^2 = c^2$, and so $(cx)^2 + (cy)^2 + (cz)^2 = c^2$. But if I scale the vector (x, y, z) by c^2, I'll get the vector (cx, cy, cz), and if S^2 were a vector space, then $(cx, cy, cz) \in S^2$. We we just checked that $(cx)^2 + (cy)^2 + (cz)^2 = c^2 \neq 1$ since $c \neq 1$. So S^2 is not a vector space with the given operations. "

   Okay, that was a little verbose. But check out the axiom that is being studied, it's the closure under scalar multiplication. This just says that if $\vec{v}$ is ANY VECTOR, and c is ANY REAL NUMBER, then $c\vec{v}$ is back again in the vector space. So to show this isn't the case we could simply choose a vector and a scalar that violates this. So a shorter answer would be:

"Let $\vec{v} = (1, 0, 0) \in S^2$ (notice $1^2 + 0^2 + 0^2 = 1$). Then $2\vec{v} = (2, 0, 0)$. But this is not in S^2 since $2^2 + 0^2 + 0^2 = 4 \neq 1$."

See how these things go? So if you want to prove something ALWAYS HAPPENS, then you really do have to read carefully the *quantifiers*: these are the words like "for all" and "there exist". It's important to know the difference. For example, if the closure property discussed above read "there exists a vector $\vec{v}$ and a scalar c so that $c\vec{v} \in V$, then all you'd have to produce is a single vector and a single scalar for such an axiom to hold! Most of the time we want a nice property like that to hold FOR ALL vectors and scalars. Of course, the "there exist" statements are useful too. For example, in a vector space **there exists** a zero vector $\vec{0}$ so that **for all** $\vec{v} \in V$ we have $\vec{v} + \vec{0} = \vec{v}$.

I hope this makes sense, but do ask about it if you need further assistance. And, as always, ROCK ON!