Math 331 Quiz #1 Review Sheet

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Hi everyone... this is Jason from the show that used to be on Cartoon Network called Home Movies. I've been hiding out in Corey's office while my parents have been in Tunisia– I know Corey is a big fan of the show and when I stopped by a few weeks ago to visit I guess I never left. Now Corey doesn't talk to me much anymore. That's bad news I guess, but the good news for you all is that I have heard a lot about what goes on and have spoken with Corey about the quiz. I've typed out below what I think is important, section by section. I hope it helps! I brought along my ferret puppets to help explain some of these concepts, as shown in the picture... I'm pretty sure they've taken this class before. They're discussing vector spaces right now. Rock on!

1. Sections 1.1 – 1.2: Systems of linear equations. In these sections we reviewed systems of linear equations and talked about why they are important. We discussed the notion of a *consistent* system and how there can only be zero, one, or infinitely many solutions to such systems. You should really know how to row reduce matrices and be very proficient at expressing your solutions. For example, once you're done row reducing a matrix (that represents a system of linear equations) you should know what the solution set looks like based on your row reduction. If there is a row of zeros except for one 1 in the augmented piece of the matrix then you know there are no solutions. If you have a free variable (and no rows like the ones I just mentioned) then there will be infinitely many solutions. If, when you're done, you can read off what $x_1, \ldots$ must be, then you must have exactly one solution. Being able to understand solutions in this way is an important part of this class later on, and I really think Corey wants you to be proficient at this material. See, for example, 13-32 in Section 1.2.

2. Sections 2.1–2.2: Matrices and their properties. In these sections we reviewed how matrices act among themselves. This includes how to add matrices of the same size and how to multiply matrices of correct size. These two properties make the set of (correctly sized) matrices a sort of algebraic system, and you should know how to find your way from place to place there. In particular, it is essential that you know how to preform the operations of multiplication and addition, and scaling by real numbers. In addition, the transpose operation will be of interest as well. The properties of matrix algebra are summarized in Section 2.2, and you should be very familiar and comfortable with these properties.

3. Section 2.3: The inverse of a matrix. (See the review for section 3.3... the inverse is discussed there).

4. Section 2.4: Elementary matrices. These will not be an important part of the class, but you should know what these matrices are and how they were used in subsequent proofs. A great example of this is how they were used in the proofs of theorems found in Chapter 3. See Tuesday's class notes for examples, but I wouldn't spend a lot of time worrying about this section right now. Instead, I would just remember that elementary row operations are really the same as multiplication (on the left) by an elementary matrix.

5. Section 3.1: Determinants. In this section we review some basic information about determinants. Some important basic facts are that you may expand a determinant about any row or column... you just need to keep the signs correct as you go. This is called expansion by cofactors along a row or column, and it makes our task of evaluating the determinant of a matrix that much easier. Imagine a matrix with a row of all zeros–expanding by cofactors along that row proves that the determinant of such a matrix is always zero. Being able to compute the determinant of a given matrix is an absolutely essential skill and I would bet my ferret Pepopo that Corey will ask a question about it.

6. Section 3.3: Properties of Determinants. This section continues where section 2.3 left off. Section 2.3 sees matrices as functions: $A : \mathbb{R}^m \to \mathbb{R}^n$. So we may ask if these functions have inverses–i.e., are they one-to-one and onto? We saw that the only matrices that have a hope of achieving both are square matrices. The rest is a crap shoot until you get to Section 3.3. Along the way we pick up important facts about inverses, for example $(AB)^{-1} = B^{-1}A^{-1}$. We also see that if A is an invertible matrix, then the linear system $A\vec{x} = \vec{b}$ has a unique solution, namely $\vec{x} = A^{-1}\vec{b}$ (just multiply on both sides by A^{-1}). There are three essential theorems in section 3.3 that you need to know very well. The first is that $\det(AB) = \det(A)\det(B)$. The second is that $\det(A) = 0$ if and only if A is singular (i.e., non-invertible). The third is that if A is invertible then $\det(A^{-1}) = \frac{1}{\det(A)}$.

7. Sections 4.1 and 4.2: Vector spaces. These sections introduce the mathematical objects known as vector spaces, the centerpiece of our study. It is introduced via your most familiar examples of $\mathbb{R}^n$. Suppose I give you a set V, a notion of addition in that set, and a way to multiply the objects in the set by scalars. Then you should be able to tell whether or not what I gave you is a vector space. If you can get by with not memorizing properties 1–10 that appear on page 186 (or as part of the class notes) then that's great. But there will be a question that is similar to 13–29 in Section 4.2, and so I suggest that you study this list very carefully. The best part about this class, though, is that almost all of the vector spaces that we'll study can be thought of as $\mathbb{R}^n$ (but not necessarily *equal* to $\mathbb{R}^n$). We'll see *exactly* how later on, but it's equally important that you understand that not all vector spaces look like $\mathbb{R}^n$–see the classnotes and book for examples of such objects. The last important notion in this section is that of a linear combination of vectors. This will be revisited time and time again, and I would know how to check whether or not a given vector is a linear combination of a given set of vectors, in addition to knowing what the term "linear combination" means. See problems 27–32 and 35–38 for examples of sample quiz questions.

8. General suggestions for the quiz. I know it seems like we've covered a lot of sections, but a lot of the concepts in those sections were tied together: finding the inverse of a matrix and row-reduction for instance. Also, there will be problems that are not solely computational–take the true/false questions and theoretical questions that appear as higher numbered homework questions. Corey will likely make these questions some part of every quiz and test. He told me he'd hate himself if he didn't. Also, for the quiz, focus on the homework questions: they should provide direction to you if you're not sure what to study. For instance, if you find yourself not being able to do a certain type of homework question, then you should work on those types of problems. Proofs will be a part of this class, and so (as I said above) I suggest that you also concern yourself with trying some of the problems that appear as higher numbered homework questions. Be sure, also, to ask for help if you're not sure what to study, or if you're confused with any concept, Corey is there to help. But above all: ROCK ON!!!