

Exam #1 solutions

Superman

February 5, 2007



Hello fellow citizens! This is Superman coming from Metropolis to give you the solutions to your recent exam. Below is the grade breakdown. ROCK ON!

Score:	Number of people who scored that:
$90 - \infty$	11
$80 - 89$	5
$70 - 79$	4
$60 - 69$	1
$-\infty - 59$	1

1. For this problem, subtract twice the first equation from the second, and this will reveal $4x_1 = 12$, so $x_1 = 3$. Plugging this into either equation gives us $x_2 = 7$.
2. Most everyone was able to do this, so I'll just list the answers. Be sure to ask Corey if you need more explanation. (a) $\begin{bmatrix} 22 & 38 \\ 14 & 23 \end{bmatrix}$, (b) $\begin{bmatrix} 8 & 14 \\ 23 & 37 \end{bmatrix}$, (c) 1, (d) -26, (e) -26, (f) $\begin{bmatrix} 5 & -8 \\ -3 & 5 \end{bmatrix}$.
3. This is not a vector space. It is not closed under addition of vectors. In particular, in the 2×2 case we see that

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \notin V.$$

4. Yes, $1 \cdot \begin{bmatrix} 6 \\ 5 \end{bmatrix} + (-3) \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

5. These vectors are linearly dependent. To show this, I need to give you a linear combination of them which is zero, without all of the coefficients being zero. Solutions to

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} a_1 + \begin{bmatrix} 1 \\ 1 \end{bmatrix} a_2 + \begin{bmatrix} 2 \\ -1 \end{bmatrix} a_3 = 0$$

translates into solving the linear system of equations with the augmented matrix below.

$$\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}.$$

So we have a_3 as a free variable, and $a_2 - a_3 = 0$. So $a_2 = a_3$, and $a_1 + a_2 + 2a_3 = 0$. So there are infinitely many solutions to this system, and all I need to do is provide a solution so that not all of a_1, a_2 and a_3 are zero. So how about $a_1 = -3, a_2 = 1$, and $a_3 = 1$.

6. I'll give solutions to each of these, in case you wanted to see the others that you didn't do.

- (a) Suppose S is the finite set of vectors $\{0, u_1, \dots, u_k\}$. Then the linear combination $a_0 \cdot 0 + a_1 \cdot u_1 + \dots + a_k \cdot u_k = 0$ when $a_0 = 1$ and $a_1 = \dots = a_k = 0$. This demonstrates that S is a dependent set of vectors.
- (b) Suppose A^{-1} is the inverse to A (we know this to exist since we are told A is invertible). Then $AA^{-1} = I$, the identity matrix, and so $\det(AA^{-1}) = \det(I) = 1$. But $1 = \det(AA^{-1}) = \det(A) \det(A^{-1})$. Since A is invertible, $\det(A) \neq 0$, so we may divide by it. Thus $\frac{1}{\det(A)} = \det(A^{-1})$.
- (c) The many hypotheses help us out a lot here. We repeatedly use $AB = BA$. For another method of proof I have given you the unneeded hypothesis that $A^2 = A$, in hopes that it would give you more ways to find a solution. $(AB)^3 = ABABAB = AABBAB = AABABB = AAABBB = A^3B^3$.

ROCK ON!