

# Math 270 Quiz # 2 Review Sheet

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*Hi kids! NARF! It's Pinky here to fill you in on all of the details for your upcoming Math 270 quiz #2. Just a reminder, it's on Thursday, February 22nd in class, and it'll be fun, fun, silly willy!!! Below I've listed the relevant sections that will be covered. You see, I've been studying this stuff, too. In the picture I am studying the effect of changing the initial conditions given a certain system of differential equations. See, I supplied the force of throwing these objects in the air (while my skeptical friend The Brain looks on), and once I let them go their path is determined by a system of differential equations. If I change their initial conditions, then I'll change their ending location. That's why a coin flip seems random. It's really not, but the smallest change in initial conditions will effect whether or not it's heads or tails. POIT! See, I'm not as stupid as I always seem on TV! Oh, and ROCK ON!*

1. Section 2.8: Dependence of Solutions on Initial Conditions. This short section is just one of those things that Corey won't let you get out of this class without seeing. It describes how the unique solution to a differential equation will change as you vary the initial condition. ZORT! It's a really interesting section, and what it says is that the function that inputs initial conditions and outputs the unique solution given those conditions is continuous: nearby initial conditions will yield nearby solutions. As much as Corey would like to ask you questions about this section, he's told me that he won't be asking any questions about this section on the quiz or test.

2. Section 2.9: Autonomous Equations and Stability. This really was a great section. We studied *autonomous* differential equations which are differential equations that don't depend on the independent variable  $t$ . We didn't really want to always solve these, because often times that's too much work for what we really want to do. So we want to uncover interesting information about the solutions if we can. If you'll recall, in class Corey went over a number of topics, all of which are fair game for the quiz and test. First, he talked about *equilibrium* solutions: these are solutions that are constants. For example, in the autonomous differential equation  $y' = (y - 1)(y - 2)$ , there are two equilibrium solutions,  $y(t) = 1$  and  $y(t) = 2$ . Once we know what the equilibrium solutions are, we can describe them by asking the question: do solutions near the equilibrium solutions tend towards the equilibrium, away from it, or neither? The ones that head towards are called "asymptotically stable". The ones that tend away are called "unstable", and everything else we really don't have a name for yet. All of this information helps us draw a possible direction field for the solutions of an autonomous differential equation. I would study the homework from this section, especially 15–20 and 23–26. I can see Corey asking you to find equilibria and to classify it as either asymptotically stable, unstable, or neither. I could also see him asking you to produce a direction field based on these results.
3. Section 3.1: Modeling Population Growth. Recall that there really were two models to study. The first was the "Malthusian" model, that  $P' = rP$ , where  $P$  is the population at time  $t$  and  $r$  is a constant measuring a society's reproductive rate. We saw this to be accurate for short periods of time, but not in the long run... after all, resources like food and space are not infinite. So a more realistic long-term model is the logistic model:  $P' = rP(1 - P/K)$ , where  $K$  is the carrying capacity. I would guess that if Corey was going to ask you about this, he would give you the differential equation I just gave you and tell you enough information to uncover the constants involved. See, for example problem 12 from this section. Even though it doesn't tell you the logistic equation formula the question looks like something Corey would ask. Or maybe not. FJORD!
4. Section 3.3: Personal Finance. I really have the same remarks about this section with regards to the types of questions Corey could ask. I can see him telling you information about a loan such as the annual interest rate and the term of the loan and asking you to find out what the monthly payments should be so that you've paid off the loan when you're supposed to have paid it off. Or maybe not. TROZ! My friend Brain had to ask: "Pinky, what is troz?" He's really smart, and wants to take over the world, and he didn't know. Do you?
5. Section 4.1: Definitions and Examples of Second Order Differential Equations. This section was awesome! What a great way to start off studying differential equations of the second order. We started off with an existence and uniqueness theorem about

linear 2nd order differential equations. Then we learned about linearly independent pairs of functions and how they fit into the general theory of finding a general solution of the homogeneous second order differential equation  $y'' + py' + qy = 0$ . I would know how to express the general solution to a 2nd order homogeneous differential equation. It's VERY important that you understand how to do this, so I would bet Corey would ask you something like: "Show that NARF is a fundamental set of solutions to ZORT. Find the general solution, and find the unique solution satisfying TROZ."

6. General suggestions. We will have covered Section 4.3 by the end of class on Thursday 2/15, or perhaps we'll finish the last bit on Tuesday 2/20 depending on how comfortable everyone is with complex numbers. You should expect at least Section 4.3 to be on the exam, but the quiz will not cover anything past 4.1, or before 2.8. Of course, that doesn't mean that you should go sniff enough glue to completely forget what you've learned this term... of course, you'll need that information as framework for this new information. And I don't think sniffing glue is good for you. We'll have some time on Tuesday to review for the quiz, so be sure to bring your questions, NARF! Oh, and

ROCK ON!