Math 270 Quiz #1 Review Sheet

By: Frylock

Hi kids. This is Frylock from Aqua Teen Hunger Force and I'm here to help you out with studying for your quiz. My other Aqua Teen Hunger Force buddies were supposed to do this review sheet for you yesterday, but that idiot Shake couldn't get his act together. Corey doesn't know this, but I float into his office whilst he writes quizzes and I've typed below the sorts of things that I thought I saw as he was typing. For each section that we've covered there is a description of what Corey finds interesting below. I would pay special attention to these interesting thoughts, and where I can, I'll try to direct you more specifically in the right direction by telling you exactly the kinds of questions to study. I hope it helps as you study for your quiz. Oh, and ROCK ON!

1. Sections 1.2-1.3: Differentiation and Integration. These sections really are meant as review from Math 211 and Math 212. Of course, you'll need a working knowledge of derivatives and integrals to be adequately prepared to do well in this class, and that's why Corey covered these the first day. If you don't feel prepared in this way (or about any other mathematical pre-requisite) then I strongly encourage you to talk to Corey about it.

2. Section 2.1: Differential equations and Solutions. This section is an introduction to differential equations and includes some new vocabulary that you must be fluent in–in general there aren't any skills that you would learn from this section, but rather you should see this section as a point at which you and Corey begin your study of differential equations, and we all need to be on the same page to start. It sort of describes the basic ideas behind differential equations, how you should study the subject, and what sorts of things are of interest in this wide field of mathematics.

I would know what it means for a differential equation to be in *normal form* (for equations of the first order, it's when you express the DE as $y' =$ something in terms of y and t). This should be something you should be able to do (because for the most part it is an exercise in algebraic manipulation).

Another key idea from this section is the idea of an *initial value problem.* This is a differential equation in which extra information is supplied that forces the solution to be unique. For instance, we know that the differential equation $y' = y$ has a family of solutions, indexed by an arbitrary constant C where $y = Ce^t$. If I supply the additional information $y(1) = 2e$, then we could solve for $C = 2$, and we would have the unique solution to the initial value problem $y = 2e^t$.

Suppose that I give you an initial value problem that has a solution, y. Then I could also ask what the domain of your solution is. You could be silly and supply for me a smaller domain than would actually work, but the largest domain of your solution is called the *interval of existance.* For example the initial value problem $y^2y' = -1$ with $y(1) = 1$ has solution $y(t) = \frac{1}{t}$. The largest domain of this function (and hence the interval of existence of this solution is $(0, \infty)$. Notice that this could possibly change if I supply different initial conditions: If $y(1) = -1$ then the solution to THAT initial value problem is $y(t) = \frac{-1}{t}$, and ITS interval of existence is $(-\infty, 0)$.

Lastly, I hope you were not asleep when Corey said that you won't be able to solve every differential equation you try. In fact, most differential equations have general solutions you won't even be able to express using elementary functions. For those that we can't, we would still like to glean some information about these solutions for future study. One way of harvesting information about possible solutions is to draw a "direction field" or "slope field". Remember, the way you would construct this is to draw a set of axes, then for each point (t, y), find out what y' must be and draw a small line with that slope at the point (t, y).

3. Section 2.2: Separable Differential Equations. This section is important, as it provides for us the first type of differential equation that we will always be able to solve. A separable differential equation is one of the form $y' = f(t) \cdot g(y)$. We write $y' = \frac{dy}{dx}$ and proceed to separate the variables:

$$y' = \frac{dy}{dx} = f(t)g(y) \Rightarrow \frac{dy}{g(y)} = f(t)dt\,.$$

Then we integrate and solve the resulting equation for y if we can. If we can't, then we'll just have to content ourselves with an "implicit solution", rather than an "explicit solution". We also discussed the meaning of a "general solution": this is a family of solutions of a differential equation that includes all but finitely many solutions. We saw that some solutions can be left out if $g(y) = 0$ for some y, and in general we will not care so much about those–at least not now. Finally, we saw that separable differential equations are essential to understanding physical phenomena such as Newton's Law of Cooling and population growth. For this section, I would definitely know how to solve any separable differential equation (problems 1–22) and know how to apply Newton's Law of cooling to some situation (see 33–35).

4. Section 2.3: Models of Motion. This section is a great example of how differential equations come about in everyday life. We studied motion on this planet both with and without air resistance. I would know that if an object traveling in a straight line has position $x(t)$, its velocity $v(t) = \frac{dx}{dt}$, and its acceleration $a(t) = \frac{dv}{dt}$. If an object acts without any forces except for the earth's gravity, then its acceleration is $a(t) = -9.8\ m/s^2$. You can use this fact to determine everything about the object if you're given enough initial data. We also entertained the notion of wind acting on an object as it travels through the atmosphere, and at this point we had to make some assumptions. Of course, that's where some of physics drifts away from math. But we assumed that the force of resistance R was a function only of v, and that it was proportional to v. Thus we determined that for some positive number r that $R(v) = -r \cdot v$. Thus since separate forces acting on an object will add to determine the total force, and by Newton's second law: $F = ma = m(-9.8) - rv$. So from there we can determine $a(t)$ and solve the differential equations that lead us to this object's position, and velocity. Unlike the previous case (without air resistance) when we solved this we found that an object's velocity is bounded by a number: it cannot exceed this speed (unless other forces act on it somewhere in the process), and it is called the terminal velocity. I would definitely be able to answer the following question about finding an object's terminal velocity: Suppose I give you the number r, a sufficient amount of initial data, and the mass of some object–can you find its terminal velocity? Corey essentially did this in class for you but should you have questions about this process, definitely ask on Tuesday. I would also know how to answer questions about motion without the influence of air resistance as well.

5. Section 2.4: Linear Differential Equations. In this section we covered a type of differential equations known as "Linear Differential Equations." They take the special form $y' = a(t)y + f(t)$. We covered two different ways of solving these equations, and I suggest that you know one of them, and be able to solve any sort of linear differential first order differential equation.

6. Section 2.7: Existence and Uniqueness. This section is really important. What is tells us is that in most situations we are garunteed a solution to the differential equation $y' = f(t, y)$, and that this solution is unique. There are two theorems to know. The first tells us when we there is a solution to $y' = f(t, y)$ at all (with initial conditions given). The second tells us when that solution is unique. I would expect you to know how to apply these theorems. For example, I suggest that you study the homework from this section (especially 25-29) to really prepare yourself. These theorems are at

the heart of the study of differential equations.

7. General info about the quiz: The quiz covers the sections listed above. The quiz will probably look a lot like the homework questions, but definitely ask if you're still not sure what I expect. Now, I'm just a talking box of french fries, but I would say that it's likely that a little more than half of the quiz will ask you to solve different types of differential equations, and the rest will be questions regarding applications, and questions from section 2.7. I hope this will point you in the right direction. So good luck, and Rock on!