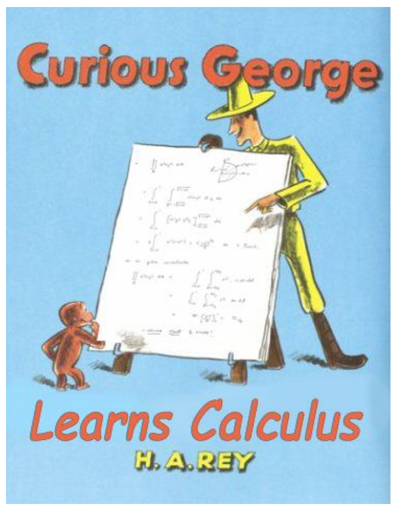
# Math 270 Midterm #2 B Solutions

Curious George and his knowledge of Calculus

March 5, 2007




*Hi kids! I've graded your Math 270 exams, and overall they looked great! Here are the solutions!* ROCK ON!

1. (a) The equilibrium solutions are $y = 2, 1, -1$.

   (b) The solutions $y = -1, 2$ are unstable, while the solution $y = 1$ is asymptotically stable.

   (c) The graph is very similar, and for some reason I can't type it out. See Corey for more information about this graph if you need it.

2. We begin from the differential equation $\frac{dP}{dt} = .1P + 1000$.

   (a) We separate variables to get

$$\frac{dP}{.1P + 1000} = dt,$$

   then integrate to get

$$10 \ln(.1P + 1000) = t + C.$$

We relabel the constant $C$ several times as we solve for $P$:

$$\begin{aligned}
\ln(.1P+1000) &= .1t + C \\
.1P + 1000 &= e^{.1t+C} = Ce^{.1t} \\
.1P &= Ce^{.1t} - 1000 \\
P &= Ce^{.1t} - 10000
\end{aligned}$$

(b) We impose the condition $P(0) = 4000$ to find $C$ as written above:

$$4000 = C - 10000 \Rightarrow C = 14000\,.$$

So $P(t) = 14000e^{.1t} - 10000$, and so $P(4) = 14000e^{.4} - 10000 \approx 10885.54$.

3. Checking that $y_1$ and $y_2$ are solutions are straightforward:

$$\begin{aligned}
y_1' &= 2e^{2t} & \qquad y_2' &= 8e^{8t} \\
y_1'' &= 4e^{2t} & \qquad y_2'' &= 64e^{8t}
\end{aligned}$$

So we can substitute now to see that $y_1'' - 10y_1' + 16y_1 = 0$, and $y_2'' - 10y_2' + 16y_2 = 0$.

(b) These are linearly independent since one is not a constant multiple of the other. You could also show that the Wronskian never vanishes as well.

(c) Set $y = C_1y_1 + C_2y_2 = C_1e^{2t} + C_2e^{8t}$. Then $y' = 2C_1e^{2t} + 8C_2e^{8t}$. So

$$\begin{aligned}
y(0) &= 0 = C_1 + C_2 \\
y'(0) &= 1 = 2C_1 + 8C_2
\end{aligned}$$

This solves as $C_1 = -1/6$ and $C_2 = 1/6$. So the answer is $y = -1/6e^{2t} + 1/6e^{8t}$.

4. (a) The roots of the characteristic equation $\lambda^2 - 2\lambda + 10 = 0$ are $\lambda = 1 \pm 3i$. You could use the quadratic equation to find this.

(b) A fundamental set of solutions is $z_1 = e^{(1+3i)t}$ and $z_2 = e^{(1-3i)t}$. Should you want real valued solutions, you can compute

$$Re(z_1) = e^t\cos(3t), \text{ and } Im(z_1) = e^t\sin(3t)\,.$$

(c) The unique solution I give will be using $z_1$ and $z_2$. If you'd like to see a solution with regard to the sin's and cos's above, then just stop by Corey's office and he'll help you out.

Set $y = C_1z_1 + C_2z_2 = C_1e^{(1+3i)t} + C_2e^{(1-3i)t}$. Then

$$y' = (1+3i)C_1e^{(1+3i)t} + (1-3i)C_2e^{(1-3i)t}.$$

So

$$\begin{array}{rcccc} y(0) & = & 0 & = & C_1 + C_2 \\ y'(0) & = & 2 & = & (1+3i)C_1 + (1-3i)C_2 \end{array}$$

This solves as $C_1 = \frac{1}{3i}$ and $C_2 = \frac{-1}{3i}$. So the answer is

$$y = \frac{1}{3i}e^{(1+3i)t} - \frac{1}{3i}e^{(1-3i)t}.$$