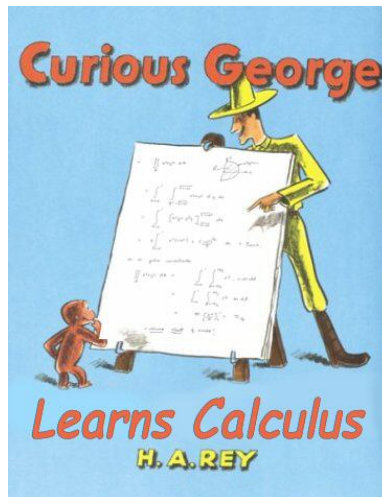


Math 270 Midterm #2 A Solutions

Curious George and his knowledge of Calculus

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Hi kids! I've graded your Math 270 exams, and overall they looked great! Here are the solutions! ROCK ON!

- (a) The equilibrium solutions are $y = -1, 2, -2$.
(b) The solutions $y = 2, -2$ are unstable, while the solution $y = -1$ is asymptotically stable.
(c) The graph is very similar, and for some reason I can't type it out. See Corey for more information about this graph if you need it.
- We begin from the differential equation $\frac{dP}{dt} = .1P + 1000$.
 - We separate variables to get

$$\frac{dP}{.1P + 1000} = dt,$$

then integrate to get

$$10 \ln(.1P + 1000) = t + C.$$

We relabel the constant C several times as we solve for P :

$$\begin{aligned}\ln(.1P + 1000) &= .1t + C \\ .1P + 1000 &= e^{.1t+C} = Ce^{.1t} \\ .1P &= Ce^{.1t} - 1000 \\ P &= Ce^{.1t} - 10000\end{aligned}$$

(b) We impose the condition $P(0) = 2000$ to find C as written above:

$$2000 = C - 10000 \Rightarrow C = 12000.$$

So $P(t) = 12000e^{.1t} - 10000$, and so $P(4) = 12000e^{.4} - 10000 \approx 7901.90$.

3. Checking that y_1 and y_2 are solutions are straightforward:

$$\begin{aligned}y_1' &= 2e^{2t} & y_2' &= 3e^{3t} \\ y_1'' &= 4e^{2t} & y_2'' &= 9e^{3t}\end{aligned}$$

So we can substitute now to see that $y_1'' - 5y_1' + 6y_1 = 0$, and $y_2'' - 5y_2' + 6y_2 = 0$.

(b) These are linearly independent since one is not a constant multiple of the other. You could also show that the Wronskian never vanishes as well.

(c) Set $y = C_1y_1 + C_2y_2 = C_1e^{2t} + C_2e^{3t}$. Then $y' = 2C_1e^{2t} + 3C_2e^{3t}$. So

$$\begin{aligned}y(0) &= 0 = C_1 + C_2 \\ y'(0) &= 1 = 2C_1 + 3C_2\end{aligned}$$

This solves as $C_1 = -1$ and $C_2 = 1$. So the answer is $y = -e^{2t} + e^{3t}$.

4. (a) The roots of the characteristic equation $\lambda^2 - 2\lambda + 10 = 0$ are $\lambda = 1 \pm 3i$. You could use the quadratic equation to find this.

(b) A fundamental set of solutions is $z_1 = e^{(1+3i)t}$ and $z_2 = e^{(1-3i)t}$. Should you want real valued solutions, you can compute

$$\operatorname{Re}(z_1) = e^t \cos(3t), \text{ and } \operatorname{Im}(z_1) = e^t \sin(3t).$$

(c) The unique solution I give will be using z_1 and z_2 . If you'd like to see a solution with regard to the sin's and cos's above, then just stop by Corey's office and he'll help you out.

Set $y = C_1z_1 + C_2z_2 = C_1e^{(1+3i)t} + C_2e^{(1-3i)t}$. Then

$$y' = (1 + 3i)C_1e^{(1+3i)t} + (1 - 3i)C_2e^{(1-3i)t}.$$

So

$$\begin{aligned}y(0) &= 0 = C_1 + C_2 \\y'(0) &= 2 = (1 + 3i)C_1 + (1 - 3i)C_2\end{aligned}$$

This solves as $C_1 = \frac{1}{3i}$ and $C_2 = \frac{-1}{3i}$. So the answer is

$$y = \frac{1}{3i}e^{(1+3i)t} - \frac{1}{3i}e^{(1-3i)t}.$$