Solutions to Exam #1

By: The Extra Credit Fairy

February 5, 2007

Hi kids! You know, it's not often that I come out from Fairyland for a visit. But sometimes Corey just makes me so mad I have to show up at his office and have a stern discussion with him. I know I look like a nice fairy, but I have a mean streak when people upset me. This time it seems that Corey asked you a question that he didn't realize would be cause for confusion. He and I had a talk about it. He really hates giving extra credit but I convinced him that there was a place on the exam that could have been done a little differently, so we agreed that I would wave my magic wand over the stack of exams and give 2 points back to each exam. I'll explain more when I get to the solution to problem 4. Below is the grade breakdown. ROCK ON!

Score:	Number of people who scored that:
$90 - \infty$	11
$80 - 89$	5
$70 - 79$	1
$60 - 69$	3
$-\infty - 59$	1

1. (a) This is a seperable DE, and so we write $4y^3y' = 3t^2 + 1$, and then integrate to get $y^4 = t^3 + t + C$. This is an implicitly defined solution, and we can square root

to get $y = \pm\sqrt{t+C}$. Some people forgot the square root, and to those I ask what would be the solution if $y(0) = -1$? Without the $\pm$, you couldn't find a solution to that, even though the existance (and uniqueness) theorem applies.

(b) With the initial condition, $C = -2$, and the interval of existance is defined as $t^3 + t \geq 2$. This is a little hard to nail down exactly, so Corey gave full credit to anyone who indicated that they knew something about this problem's interval of existance.

(c) With the initial condition, $C = -1$, and the interval of existance is $t^3 + t \geq 1$. Same: This is a little hard to nail down exactly, so Corey gave full credit to anyone who indicated that they knew something about this problem's interval of existance.

2. We use the variation of parameters for this one. If you do it with the u's, then come see me and I'll show you another solution.

   First, we solve $y_h' = \frac{4}{t}y_h$. This has a solution $y_h = t^4$. Set $y = vy_h$. Then after you work out $y' = v'y_h + vy_h'$ and $\frac{4y}{t} + t^3 = \frac{4vy_h}{t} + t^3$, we get $v'y_h = t^3$, or $v' = \frac{1}{t}$. So the general solution for $v = \ln t + C$, so $y = t^4(\ln t + C)$.

3. We start with $\frac{dT}{dt} = k(T - 50)$, we're given $A = 50$ in the problem. This seperable differential equation solves as $T(t) = Ce^{kt} + 50$. Using $T(0) = 100$, we get $C = 50$, and using $T(1) = 80$:

$$T(1) = 80 = 50e^k + 50 \Rightarrow \frac{3}{5} = e^k \Rightarrow k = \ln\frac{3}{5}\,.$$

   So we want to know, for what t, do we have $T(t) = 65$. So we use the information we have uncovered to get

$$T(t) = 65 = 50e^{\ln\frac{3}{5}t} + 50 \Rightarrow \frac{15}{50} = e^{\ln\frac{3}{5}t} \Rightarrow t = \frac{\ln\frac{15}{50}}{\ln\frac{3}{5}} \approx 2.35\ldots\,.$$

4. Here is where I got really upset with Corey. I'll start out with the solution and point out what it was I didn't like. I got Corey to admit, after the exam but before he graded them, that he could see something he wasn't happy with. I'll point out that, too.

   (a) If $a(t) = -300e^{-t}$, then the velocity is $v(t) = \int a(t)dt = 300e^{-t} + v_0$. Now, since you dropped the object, we have the initial condition $v(0) = 0$. So at this point a lot of you decided that $v_0 = 0$. But this velocity equation doesn't work like that:

$$v(0) = 0 = 300e^0 + v_0 = 300 + v_0 \Rightarrow v_0 = -300\,.$$

   It was Corey's intention to ask you a question where you had to demonstrate your understanding of the relationship between acceleration, velocity, and position. To his credit, he also wanted to give you an equation that would be easy to integrate. And quite frankly, everyone should have known that $v(0) = 0$ is the right initial condition to impose, and that from there one is to find the actual value of v_0. But after a loud and heated discussion I finally convinced Corey that this is a little misleading by pointing out that all of the examples you've done with the same initial condition have resulted in $v_0 = 0$. And so it's no wonder that almost none of you found the correct value of $v_0 = -300$.

Now, in general, Corey isn't stupid, and he sort of realized this right away before he started grading the exams. And so he decided to take it easy on the class as a whole on this problem, and for those people who didn't find the correct value of v_0 but did everything else right that he would take off 2 points out of 10. On the next part, if there was an indication that you knew how to find the correct answer to part (b), regardless if you found the right value for v_0 in part (a), that you would get full credit. Of course, there were people who made mistakes outside the scope of these matters that were penalized for other reasons.

So Corey and I agreed that I would wave my wand over the exams and put back the 2 points on every exam, no matter if you made this mistake or not. Even though Corey initially graded this problem pretty easily anyway, it still gets under my skin that he asked a question that was slightly misleading–even though it was unintentional. After finishing with this document I have to hustle back to Fairyland to pick up my Fairy Social Security check, so direct any other questions to Corey in his office.

5. Let $f(x,y) = y^{1/3} + \frac{y}{x-1}$.

   (a) The answer is no, we don't know for sure. For existance, there are two things to check, but before you can check them, you need a rectangle R. You need to check that (1) f is continuous on R, and (2) your initial condition is a point in R. So let $R = (1,\infty) \times \mathbb{R}$. Really, unless you give me a rectangle there is still something missing from your solution, keep that in mind for the future.

   (b) The answer is no. For the uniqueness we have to check 3 things. The first two are the two above in existance. The next one is that $\frac{\partial f}{\partial y}$ is also continuous on R. But you'll notice that $\frac{\partial f}{\partial y} = \frac{1}{3}y^{-2/3} + \frac{1}{x-1}$ is not continuous when $x = 1$ or when $y = 0$ (look at the first term $y^{-2/3}$). So we don't know for sure.

Rock on!