Math 251 Quiz # 3 Solutions

The Japanese word for agony, Kumon!

March 17, 2008

Kumon!

1. (a) $\nabla f = (e^v, \cos v + ue^v)$, (b) At $(0,0)$, the gradient is $\nabla f|_{(0,0)} = (1,1)$. So the unit vector in that direction is $\frac{1}{\sqrt{2}}(1,1)$. (c) The directional derivative is defined as $\nabla f \cdot \vec{v} = (1,1) \cdot (1,2) = 3$. (d) This composition has a partial derivative

$$\begin{aligned} \frac{\partial h}{\partial y} &= \frac{\partial f}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} \\ &= e^v(1) + (\cos v + ue^v) \cdot (4y) \\ &= e^{2y^2} + (\cos(2y^2) + (x+y)e^{2y^2})(4y). \end{aligned}$$

2. Using the function $g(x,y,z) = x^2 - xy - z$, we notice that the gradient ∇g is normal to the level curve $g = 0$. Now $\nabla g = (2x - y, -x, -1)$, and at the point $(1,-1,2)$, $\nabla g|_{(1,-1,2)} = (3,-1,-1)$. So the equation of the tangent plane is determined by the equation $0 = (\nabla g) \cdot (x - 3, y + 1, z - 2) = 0$, or $3(x-1) - (y+1) - (z-2) = 0$, or $3x - y - z = 2$.

3. (a) The derivative $\gamma'(t) = (1, 2, 2t)$ and so $\gamma'(1) = (1,2,2)$.

   (b) The equation of the tangent line is $\vec{c}(t) = (1,2,1) + t(1,2,2)$.