Quiz # 2 Solutions

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Mmmmm. Yeah. Alright, I'm going to go ahead and give you the solutions to your quiz # 2, mmmmmkay? Then I'm going to go ahead and ask that you move your desk to storage room B, and take care of the rat problem down there. Mmmmkay... thanks!

1. (a) We compute $\vec{a} \times \vec{b}$ using our symbolic determinant notation:

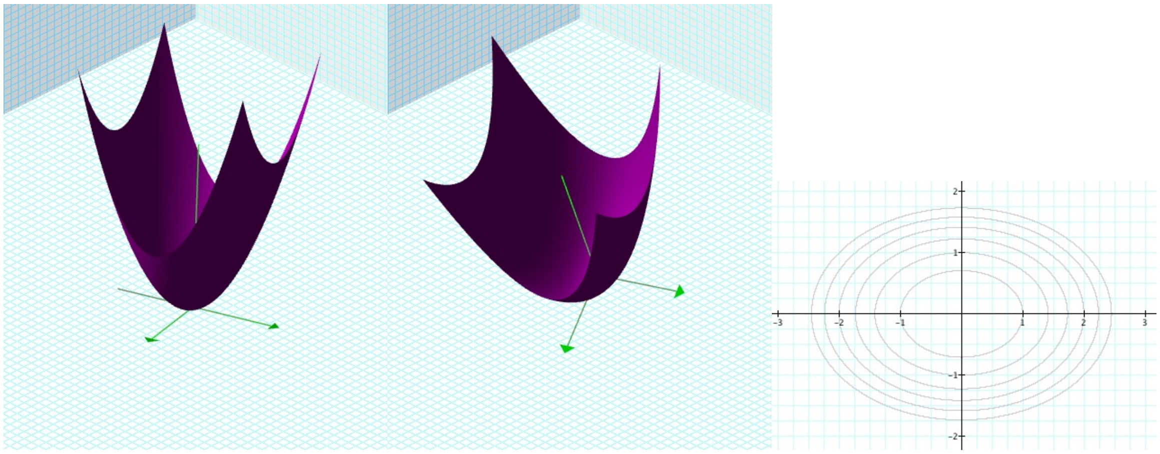
$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 0 & -2 & 4 \end{vmatrix} = \vec{i}(4-(-2)) - \vec{j}(4-0) + \vec{k}(-2-0) = 6\vec{i} - 4\vec{j} - 2\vec{k}.$$

(b) The area of the parallellogram spanned by these vectors is the length of the cross product above:

$$||\vec{a} \times \vec{b}|| = \sqrt{6^2 + (-4)^2 + (-2)^2} = \sqrt{56}.$$

(c) The volume of this parallellopiped is given by the quantity $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = 6$. Of course, one could also compute a determinant with rows $\vec{a}, \vec{b}$ and $\vec{c}$, and this could be done in any order of rows, as the absolute value is taken after the determinant is done, and exchanging the order of the rows only introduces a change of sign to the determinant.

2. Here are several views of the graph of $f(x, y) = x^2 + 2y^2$. The first two are two different perspectives on the same graph, the x-axis is coming out at you. The last are several level curves for $c = 0, 1, 2, 3, 4, 5, 6$ (note that there are no level curves to draw for $c < 0$:



3. We change the coordinate using the conversions found on page 69 of our book, or in your notes:

$$\begin{aligned} x &= 5\sin(\pi/3)\cos(\pi/6) &&= \tfrac{15}{4} \\ y &= 5\sin(\pi/3)\sin(\pi/6) &&= \tfrac{5\sqrt{3}}{4} \\ z &= 5\cos(\pi/3) &&= \tfrac{5}{2}. \end{aligned}$$

4. No, this limit does not exist. Consider the path $x = t$, $y = 0$, heading to $(0, 0)$ as $t \to 0$. Then this function is identically 0, and thus, if the limit exists then it must be 0. But as you approach $(0, 0)$ along the path $x = t, y = t$, then we see that the function along that path is identically $\frac{1}{2}$, and so since the limit is unique, and must be two different values, it must not exist. Below is a graph of the function $f(x, y) = \frac{xy}{x^2+y^2}$: