Quiz # 1 solutions

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Hello kids. As you can see, I'm a talking broccoli crown who likes to lift weights. I'm thinking of running for governor of the state of California. And as part of my campaign, I'm providing to you the solutions to the quiz so that I might gain your vote! ROCK ON!

1. (a) $(6, -12, 18)$
   (b) $(0, -8, 12)$
   (c) $\sqrt{2^2 + (-4)^2 + 6^2} = \sqrt{56}$
   (d) $\sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$
   (e) $\vec{u} \cdot \vec{v} = 2 \cdot 1 + (-4) \cdot 2 + 6 \cdot (-3) = -24$
   (f) The cosine of the angle between these vectors is $\frac{-24}{\sqrt{56}\sqrt{14}}$

2. First we find which direction the line heads in. The vector from P to Q is exactly what we want, and that vector is $\vec{v} = Q - P = (0, 1, 3)$. Thus the line starting at P and heading in direction $\vec{v}$ is $\ell(t) = P + t\vec{v} = (1, 2, 3) + t(0, 1, 3) = (1, 2 + t, 3 + 3t)$.

3. First we will find what the equation of the line is, and then determine if they intersect. The new line has equation $\gamma(t) = (1,0,0) + t(0,1,2) = (1,t,2t)$. If they intersect, then there are $t_1, t_2 \in \mathbb{R}$ so that $\ell(t_1) = \gamma(t_2)$. To phrase it in coordinates, we have that

$$\begin{aligned} 1 &= 1 \\ 2 + t_1 &= t_2 \\ 3 + 3t_1 &= 2t_2. \end{aligned}$$

Substituting the second equation into the right side of the third, we have $3 + 3t_1 = 2(2 + t_1) = 4 + 2t_1$. Or, $t_1 = 1$. Going back to the second equation, we have $t_2 = 2 + t_1 = 2 + 1 = 3$. So these lines intersect.