

Math 251 Midterm # 2 Solutions

Evie Mae Dunn

March 5, 2008

Hi everyone! I'm Corey's new niece, and I was born yesterday, March 4th, 2008! I sort of had a busy day yesterday at the hospital, and entertaining family. But today I managed to get away for a little bit and write your solutions today. Finally! Some alone time! And, of course, mathematics would be my choice of activity. ROCK ON!

1. (a) We compute the cross product using our symbolic determinant:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 4 & 1 \\ 1 & 2 & 4 \end{vmatrix} = \vec{i}(16 - 2) - \vec{j}(0 - 1) + \vec{k}(0 - 4) = 14\vec{i} + \vec{j} - 4\vec{k}.$$

(b) The area is the length of the cross product: $\|\vec{a} \times \vec{b}\| = \sqrt{14^2 + 1^2 + 4^2} = \sqrt{213}$.

(c) The volume is the absolute value of the dot product between the vectors shown: $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |(14, 1, -4) \cdot (1, 1, 0)| = 15$.

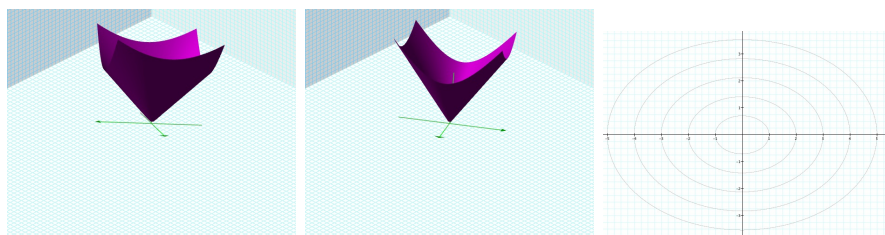
(d) The plane has normal vector $\vec{n} = (14, 1, -4)$ and goes through the point $(1, 0, 0)$, so the equation is

$$14(x - 1) + 1(y - 0) + (-4)(z - 0) = 0 \iff 14x + y - 4z - 14 = 0.$$

(e) The distance is given by the formula

$$\frac{|14(0) + 1(0) + (-4)(0) - 14|}{\sqrt{14^2 + 1^2 + 4^2}} = \frac{14}{\sqrt{213}}.$$

2. Here are some views of the graph, and of some level sets. Keep in mind that the program that views these graphs is displaying the surface above a rectangular region, giving the appearance of 4 "points" to the surface. . . this is purely an artifact of the technology used to produce these graphs.



3. We use the change of coordinate transformations $x = r \cos \theta = 3 \cos(\frac{\pi}{3}) = 3/2$,
 $y = r \sin \theta = 3 \sin(\frac{\pi}{3}) = \frac{3\sqrt{3}}{2}$, and $z = z = 2$.
4. This limit does not exist. We discussed this at length in class after the exam, consider the path $x = 0, y = t$, and $x = 2t, y = t$. These paths give you two different limits, thus this limit must not exist.
5. For $f(x, y) = e^{xy} + \sin y$, we have

$$\frac{\partial f}{\partial x} = ye^{xy}, \quad \frac{\partial f}{\partial y} = xe^{xy} + \cos y.$$

6. For $f(x, y, z) = \sqrt{x^2 - 2y^2 + z}$, we have

$$\begin{aligned} \frac{\partial f}{\partial x} &= \frac{1}{2} \frac{1}{\sqrt{x^2 - 2y^2 + z}} \cdot 2x \\ \frac{\partial f}{\partial y} &= \frac{1}{2} \frac{1}{\sqrt{x^2 - 2y^2 + z}} \cdot (-4y) \\ \frac{\partial f}{\partial z} &= \frac{1}{2} \frac{1}{\sqrt{x^2 - 2y^2 + z}} \cdot 1. \end{aligned}$$