

Math 251 Midterm # 1 solutions

The Nintendo version of Mike Tyson

February 8, 2008



Hi folks! This is the Nintendo image of Mike Tyson! I've taken a break from boxing to write up solutions to your first midterm. Generally, the exams looked good, and Corey only has comments about a few things that he'll discuss in class.

1. (a) $2\vec{v} = (0, 4, -6)$, (b) $3\vec{u} - \vec{v} = (3, -11, 9)$, (c) $\|\vec{u}\| = \sqrt{1^2 + (-3)^2 + 2^2} = \sqrt{14}$, (d) $\|\vec{v}\| = \sqrt{0^2 + 2^2 + (-3)^2} = \sqrt{13}$, (e) $\vec{u} \cdot \vec{v} = 1 \cdot 0 + (-3) \cdot 2 + 2 \cdot (-3) = -12$, so for part (f) we have the $\cos \theta = \frac{-12}{\sqrt{14}\sqrt{13}}$. The angle itself would be the inverse cosine of that quantity, and Corey accepted either answer.
2. If a line $\ell(t)$ goes through point P in direction \vec{v} , then the equation $\ell(t) = P + \vec{v}t$. So the equation of the line in this problem would be

$$\ell(t) = (1, 0, 6) + (-1, 5, 8)t = (1 - t, 5t, 6 + 8t).$$

There seemed to be some confusion with the equation of a line between 2 points and the equation of a line going in a given direction. Corey marked each of the times he encountered this confusion, but for those who had this trouble be sure to know the difference!

3. The line described in this problem has equation $\gamma(t) = (1 - 2t, 8 - 3t, 2 + 5t)$, and they intersect if there is a t_1, t_2 so that $\ell(t_1) = \gamma(t_2)$. So we compare each coordinate and conclude that $1 - t_1 = 1 - 2t_2$. Then $t_1 = 2t_2$. The next equation $5t_1 = 8 - 3t_2$, so $5(2t_2) = 10t_2 = 8 - 3t_2$, or $t_2 = \frac{8}{13}$. So $t_1 = \frac{16}{13}$. Checking these values in the last equation $6 + 8t_1 = 2 + 5t_2$ yields a contradiction. So these lines don't intersect.
4. For (a) and (b) of this problem, one can verify the Cauchy-Schwartz and triangle inequality hold. For part (c), we see that

$$\text{proj}_{\vec{v}}\vec{u} = \frac{\vec{u} \cdot \vec{v}}{\|\vec{v}\|^2} \vec{v} = \frac{-5}{18}(1, -1, 4).$$

For part (d), we notice that the vector $\vec{a} = \text{proj}_{\vec{v}}\vec{u}$, and since \vec{u} has been given to us, one computes what \vec{b} has to be as the difference $\vec{b} = \vec{u} - \text{proj}_{\vec{v}}\vec{u} = (3, 8, 0) - \frac{-5}{18}(1, -1, 4)$.

5. (a) The equation through P and Q has direction vector $Q - P = (0, -2, -2)$, so the line is $\ell(t) = (0, 1 - 2t, 1 - 2t)$. Plugging $x = 0$, $y = 1 - 2t$ and $z = 1 - 2t$ into $x^2 + y^2 - z^2$ gives a value of 0. (c) Since $m(t)$ goes through the origin and has direction vector (v_1, v_2, v_3) , $m(t) = (v_1t, v_2t, v_3t)$. Since m is perpendicular to ℓ , their direction vectors are perpendicular, so $(0, -2, -2) \cdot (v_1, v_2, v_3) = -2v_2 - 2v_3 = 0$, or that $v_2 = -v_3$. Lastly, since it's on the same surface as above, we have $(v_1t)^2 + (v_2t)^2 - (v_3t)^2 = 0$. Simplifying, we conclude $v_1^2t^2 + v_2^2t^2 - v_3^2t^2 = 0$. Since this must hold for all t , it must work for nonzero values of t and thus we may cancel the common factor of t^2 to conclude $v_1^2 + v_2^2 - v_3^2 = 0$. But we already know that $v_2 = -v_3$, so $v_2^2 = v_3^2$, and so using this in the above we get $v_1^2 = 0$, or $v_1 = 0$. So any choice for v_2 and v_3 with $v_2 = -v_3$ will give us an answer. So how about $v_2 = 1, v_3 = -1$, and $m(t) = (0, t, -t)$.