The final exam Math 251 review sheet

The Kid Problem Solver

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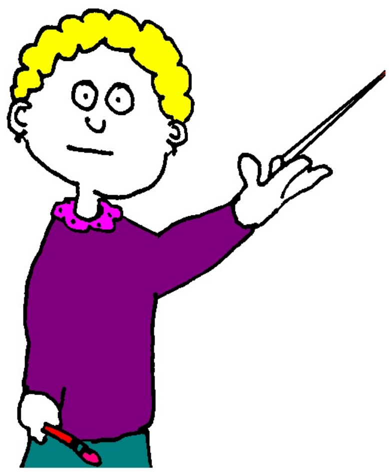

Hi everyone! Here are the last few sections to help you review for the final exam!

1. Section 3.1: Iterated partial derivatives. Really there are only two big ideas in this section. First, the observation that one can take repeated partial derivatives (analagous to taking many derivatives in single variable calculus). Second, that mixed partial derivatives commute for C^3 functions.

2. Section 3.2: Taylor series. We use the mixed partial derivatives and repeated differentiation from the last section to construct a notion of power series for functions of several variables. Corey will ask you to compute a 2nd degree Maclaurin polynomial of a function that you are familiar with on the final exam, so be sure to be versed in this method.

3. Section 3.3: Extrema. We use Taylor polynomials to motivate a discussion about finding extrema of functions of 2 variables. We note that any function attains a local min or max only at a critical point: where $\nabla f = 0$. So when one searches for extrema, one only need consider those inputs where the gradient vanishes. After that, one considers Hessians, special quadratic functions that help to determine extrema

in the following way. If A is the matrix of 2nd partial derivatives, then one may form the Hessian associated to A as described in class. From there, at whatever inputs you are interested in (critical points) if the Hessian is positive definite, then that input is a local min. If the Hessian is negative definite, then that input is a local max, and if the Hessian is indefinite, then the input is a saddle point. Recall that for the symmetric matrix with entries $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$, then A is positive definite if $a > 0$ and $ac - b^2 > 0$. A is negative definite if $a < 0$ and $ac - b^2 > 0$, and A is indefinite if only $ac - b^2 \neq 0$. Corey feels that he was not entirely clear on this in class and wanted to be very explicit about this.

So to find each extrema, one first collects the critical points. At each critical point, one finds the matrix of 2nd partial derivatives evaluated at the critical point in question. Then one proceeds to classify the critical point as a local min, a local max, or a saddle point according to the 2nd derivative test. The example done in class is a great example, as are the homework questions.