

Math 213 Quiz # 2 Solutions

By: Finals Week

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Hi kids! Finals Week here, just pointing out that I'm right around the corner. Say hi to my friend Dead Week for me! Enjoy these solutions!

1. (a) Let $a_n = \frac{x^n}{4^n n^2}$. Then using the ratio test we have:

$$\left| \frac{\frac{x^{n+1}}{4^{n+1}(n+1)^2}}{\frac{x^n}{4^n n^2}} \right| = \left| x \cdot \frac{1}{4} \frac{n^2}{(n+1)^2} \right| \rightarrow \frac{|x|}{4}.$$

So when $\frac{|x|}{4} < 1$, we have $|x| < 4$, and the radius of convergence is 4. Plugging in $x = \pm 4$ gives us the series $\sum \frac{\pm 1}{n^2}$, which converges. So the interval of convergence is $[-4, 4]$.

- (b) Using the ratio test with $a_n = \frac{(n+1)x^n}{n!}$ gives us

$$\left| \frac{\frac{(n+2)x^{n+1}}{(n+1)!}}{\frac{(n+1)x^n}{n!}} \right| = \left| x \cdot \frac{1}{n+1} \cdot \frac{n+2}{n+1} \right| \rightarrow 0.$$

So the radius of convergence is ∞ , and the interval of convergence is $(-\infty, \infty)$.

2. Notice that $f'(x) = 2e^{2x}$, $f''(x) = 2^2 e^{2x}$, $f'''(x) = 2^3 e^{2x}$, and so we conjecture that $f^{(n)}(x) = 2^n e^{2x}$, and so $f^{(n)}(0) = 2^n$. Then the Maclaurin coefficients $a_n = \frac{2^n}{n!}$, and so $e^{2x} = \sum \frac{2^n}{n!} x^n$.
3. We notice $\frac{1}{1+x^3} = \frac{1}{1-(-x^3)} = \sum_0^\infty (-x^3)^n = \sum_0^\infty (-1)^n x^{3n}$.
4. We notice that $\frac{x}{3} = \cos t$, and that $\frac{y}{3} = \sin t$, so that

$$\left(\frac{x}{3}\right)^2 + \left(\frac{y}{3}\right)^2 = \frac{x^2}{9} + \frac{y^2}{9} = 1.$$

So the graph is a circle centered at the origin of radius 3.

