Math 213 Quiz # 1 Solutions

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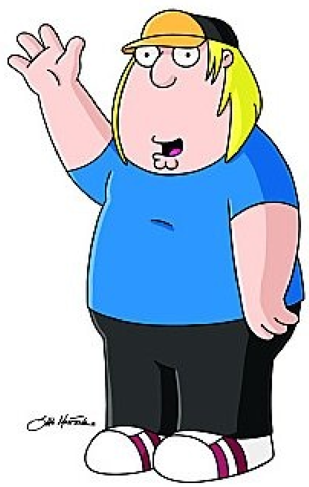

HI EVERYBODY! It's Chris Griffin here to tell you the solutions to the recent quiz # 1. HOPE YOU LIKE IT! Please see the end of the document for other general remarks about this quiz.

1. (a) This sequence converges to 3. This can be seen by factoring out an n^4 on the numerator or denominator, or by L'Hospital's Rule. Below, we use L'Hospital's Rule:

$$\lim_{n\to\infty} \frac{3n^4+1}{n^4+2n+1} = \lim_{x\to\infty} \frac{3x^4+1}{x^4+2x+1} = \lim_{x\to\infty} \frac{12x^3}{4x^3+2} = \lim_{x\to\infty} \frac{36x^2}{12x^2} = \lim_{x\to\infty} \frac{36}{12} = 3.$$

(b) We use the squeeze theorem, and note that $0 \le \sin^2(n) \le 1$. So

$$0 \le \frac{\sin^2(n)}{n} \le \frac{1}{n}.$$

Now since both $0 \to 0$ and $\frac{1}{n} \to 0$, we conclude that $\frac{\sin^2(n)}{n} \to 0$.

(c) We note that $\frac{n!}{(n+2)!} = \frac{1}{(n+1)(n+2)} \to 0$.

2. (a) This series diverges by the test for divergence:

$$\lim_{n\to\infty} \frac{n+1}{n} = 1 \ne 0.$$

(b) We use the limit comparison test with the series $b_n = \frac{1}{n}$, and $\sum \frac{1}{n}$ is known to diverge since it is a p-series, with $p = 1$. Then

$$\lim_{n\to\infty} \frac{1/(n+1)}{1/n} = \lim_{n\to\infty} \frac{n}{n+1} = 1, 0 < 1 < \infty.$$

So the limit comparison test applies, and since $\sum \frac{1}{n}$ diverges, we have that $\sum \frac{1}{n+1}$ diverges.

Remark. A surprising number of you made a direct comparison of the form $\frac{1}{n+1} \leq \frac{1}{n}$, *which is true. But since* $\sum \frac{1}{n}$ *diverges, this comparison is of no use. In a certain sense, all it would say is that* $\sum \frac{1}{n+1} \leq \infty$, *which does not force it to either converge or diverge.*

(c) This is a geometric series, as $e^{-n} = \left(\frac{1}{e}\right)^n$. So since $-1 < \frac{1}{e} < 1$, we conclude that the given series converges.

(d) We use the limit comparison test with $b_n = \frac{1}{n}$, which as mentioned before, $\sum \frac{1}{n}$ is a divergent p-series. We check

$$\lim_{n\to\infty} \frac{n/(n^2 - 4n + 18)}{1/n} = \lim_{n\to\infty} \frac{n^2}{n^2 - 4n + 18} = 1.$$

So the limit comparison test applies, and the given series diverges.

(e) We use the limit comparison test with $b_n = \left(\frac{3}{4}\right)^n = \frac{3^n}{4^n}$. The series $\sum \left(\frac{3}{4}\right)^n$ is a convergent geometric series. Then we check that the limit comparison test applies, first by studying the quotient $\frac{a_n}{b_n}$:

$$\frac{\frac{3^n}{4^n - 1}}{\frac{3^n}{4^n}} = \frac{4^n}{4^n - 1}.$$

Now we apply L'Hospital's rule to see that

$$\lim_{n\to\infty} \frac{4^n}{4^n - 1} = \lim_{x\to\infty} \frac{4^x}{4^x - 1} = \lim_{x\to\infty} \frac{(\ln 4)4^x}{(\ln 4)4^x} = \lim_{x\to\infty} 1 = 1.$$

Rock on!